

ADAPTIVE ANGLE TRACKING AND CORRELATION
FOR AIRBORNE DIRECTION-FINDING

by

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THESIS

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ABSTRACT

An airborne direction-finding technique capable of handling multiple emitters was developed. An adaptive gate size was introduced in the track correlations based on covariance relations of established tracks and observations. Track files were generated based upon the existence of various target parameters; i.e., frequency, pulse repetition frequency, pulse width and direction-of-arrival. To test the angular resolution capability of the filter, emitters in close proximity to each other with identical electronic characteristics were used in the simulation. Target locations are calculated in a cartesian coordinate system where the sphericity of the earth is taken into account, and with appropriate coordinate transformation computational simplicity is preserved.

TABLE OF CONTENTS

I.	INTRODUCTION -----	6
II.	COMPUTATIONAL PROCEDURE -----	9
	A. VARIOUS APPROACHES -----	9
	B. AN ADAPTIVE BEARING ANGLE FILTER -----	9
	C. PARAMETER CORRELATION -----	18
	D. TARGET LOCATION AND CORRELATION BY CLUSTER POINTS -----	20
III.	PRESENTATION OF RESULTS -----	23
	A. TARGET LOCATION -----	23
	B. TARGET CORRELATION -----	31
	C. STATISTICAL EVALUATION -----	34
IV.	CONCLUSIONS -----	37
	APPENDIX A -----	39
	COMPUTER OUTPUT -----	43
	COMPUTER PROGRAM -----	68
	BIBLIOGRAPHY -----	79
	INITIAL DISTRIBUTION LIST -----	80
	FORM DD 1473 -----	83

LIST OF TABLES

TABLE

I	CORRELATION RESULTS FROM 20 OBSERVATIONS (100% DETECTION OF SIX TARGETS)	32
II	CORRELATION RESULTS FROM 20 OBSERVATIONS (50% DETECTION OF SIX TARGETS)	33

LIST OF FIGURES

FIGURE

1	SIMULATION SCENARIO	13
2	MEAN ERROR OF BEARING ANGLE	14
3	MEAN ESTIMATE OF ANGULAR RATE AND RATE ERROR	15
4	STEPS FOR TARGET LOCATION BY "CLUSTER" POINTS	22
5	TARGET LOCATION BY "CLUSTER" METHOD (THREE TARGETS)	24
6	TEN RUNS OF TARGET LOCATION BY "CLUSTER" METHOD	25
7	TARGET LOCATION BY "CLUSTER" METHOD (SIX TARGETS)	26
8	TARGET LOCATION USING KALMAN FILTER (10 OBSERVATIONS)	28
9	TARGET LOCATION USING KALMAN FILTER (20 OBSERVATIONS)	29
10	TARGET LOCATION USING KALMAN FILTER (50% DETECTION)	30
11	VARIANCE OF BEARING ANGLE ESTIMATE	35
12	VARIANCE OF ESTIMATE OF ANGULAR RATE	36
13	BLOCK DIAGRAM OF KALMAN FILTER AND SMOOTHING SOLUTION	42

I. INTRODUCTION

In airborne direction-finding (DF) a good procedure for locating the emitter given angle observations only is of primary importance. This becomes very difficult in a multi-emitter environment. With sophisticated hardware and software, many of these emitters can be identified and sorted by their pulse repetition frequency (PRF), and pulse width (PW). However, without accurate direction of arrival information and methods for correlating these with successive past emissions, the problem is not solved. Central to all of this is the estimate of the location of the emitters.

In the emitter locating portion of the study, it was assumed that all emitters have the same frequency, PRF, and pulse width, and thus, except for their angle of arrival, all correlate as the same target. This is really a very difficult problem. To get an idea just how difficult, consider a rather typical environment of six emitters in reasonable proximity, and an aircraft with a noisy antenna system making ten observations of each of these emitters. The possible number of locations with a straight-forward method of locating by calculating all possible intersections of bearing lines will give 1,620 locations computed by the formula

$$m^2 n(n-1)/2$$

where m = number of emitters and n = number of observation points. The computer does not know whether there are six

emitters or 1,620. Without sophisticated association and correlation routines for correlating new observations with previous tracks and adaptive angle tracking and gating, the problem is virtually insolvable.

Two methods of solving the location and correlation problem were investigated. The first uses a Kalman filter to compute predicted values of angle of arrival based on past data, and a correlation routine to correlate the present observations with these predicted values. If no correlation was possible, it initializes a new track. A correlated observation is then used to compute an optimal estimate of the present angle-of-arrival for that track. In the second method, all data from a number of observation stations are processed simultaneously by first calculating all possible target locations based on intersections with the first bearing obtained. Then, the mean position of this cluster is found, declared a probable target location, and the observations having contributed to this location are correlated to this target. In both cases, the target location is found in an earth-centered coordinate system (where the x_3 axis is the polar axis and the x_1, x_2 plane is the equatorial plane of the earth, with the positive x_1 axis corresponding to 0° longitude and the positive x_2 axis corresponding to 90° longitude east). For each observation, an observation plane is defined, passing through the earth's center and the bearing line at the observation point, such that in the noiseless case, this plane would

also pass through the target point. The i_{th} plane is then defined by

$$a_{1i}x_1 + a_{2i}x_2 + a_{3i}x_3 = 0, \quad (1)$$

where the coefficients a_{ji} can be found from the observation station coordinates and the observation angle. The procedure for obtaining a target location is to take the equations of the observation planes, two at a time, with the constraint equation that the emitter be located at the earth's surface. Solving the equations of two planes below, gives the desired target location.

$$a_{11}x'_1 + a_{12}x'_2 = -a_{13}c \quad (2)$$

$$a_{21}x'_1 + a_{22}x'_2 = -a_{23}c \quad (3)$$

The equations were modified to solve for x_1 and x_2 with $x_3 = c$. The constraint equation is accounted for by solving for

$$E = R/(x'^2_1 + x'^2_2 + c^2)^{1/2} \quad (4)$$

where R = radius of the earth. The resulting solution for x_i is

$$x_i = E x'_i \quad i = 1, 2, 3. \quad (5)$$

By setting $x_3 = c$, the observation planes have been projected into lines on the plane $x_3 = c$, then in solving (2) and (3) the intersection of these lines in the $x_3 = c$ plane is found. The parameter E is then utilized to project this point to the surface of the earth. With proper coordinate rotation, it is possible to have the projection plane ($x_3=c$) earth surface tangent and aircraft centered.

II. COMPUTATIONAL PROCEDURE

A. VARIOUS APPROACHES

As is always the case, there are many ways to attack this problem. Possibly the best technique for evaluating the various alternatives is through a realistic statistical evaluation of a multi-emitter environment. But first a careful description of the two alternative techniques considered here, and the methods of analyzing the techniques (i.e., the simulation scenario) is in order. First, a procedure for generating the target and receiver kinematics is necessary. Then, for the multi-emitter environment, a very sophisticated association and correlation routine for relating the observations with existing tracks is required. Lastly, the location of the emitters must be found, given either estimates of bearing angles or, alternatively, a large set of noisy bearing lines giving a set of noisy locations from which a location estimate is derived.

B. AN ADAPTIVE BEARING ANGLE FILTER

A Kalman filter[1] is used to obtain estimates and prediction of bearing angles for the next observation point to allow the correlation of observations with past data. Each observation and bearing line will have an unknown angular rate associated to it. This rate depends on the speed and heading of the aircraft, the bearing itself,

and the unknown range to the emitter for fixed or slowly moving targets. It is, however, approximately known; and this approximation is used in the initialization of the filter for each target. The initial uncertainty of angle and angular rate on track initialization is accounted for through the apriori covariance of error initialization in the filter. The filter also takes into account missing data points. That is, at any instant of look, at most only 50% of the targets might be emitting. Trackfile management, like initializing a new track, dropping a track, or flagging a missing observation are done in the correlation section.

The noisy observation z is assumed to be described by

$$z_k = \underline{h}_k \underline{\theta}_k + v_k, \quad k = 1, 2, \dots \quad (6)$$

Here $\underline{h}_k = (1 \ 0)$, and $\underline{\theta}_k = (\theta_k \dot{\theta}_k)^T$ is the true observation angle and angular rate, and v_k represents a white noise sequence. Since the observer is moving in time (the subscript k denotes the k^{th} instant in time), a difference equation is used to approximate the system dynamics:

$$\underline{\theta}_k = \Phi_{k,k-1} \underline{\theta}_{k-1} \quad k = 1, 2, \dots \quad (7)$$

No random forcing is included; this implies stationary targets and exact knowledge of the observer's position. The filter can be extended to take moving targets into account by augmenting the difference equation above, and using an appropriate non-zero Q matrix when initializing the recursive filter gain and covariance equations. The mean and variance of the measurement noise sequence v_k is given by:

$$E(v_k) = 0 \text{ for all } k \quad (8)$$

$$\text{and} \quad E(v_k v_j) = R_k \delta_{jk} \quad (9)$$

$$\text{where} \quad \delta_{jk} = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$$

The recursion equations for filtering are defined below, where $\hat{\theta}_{k/j}$ denotes the estimate of the state $\underline{\theta}_k$ based upon measurement data (z_1, z_2, \dots, z_j)

$$\hat{\underline{\theta}}_{k/k-1} = \Phi_{k,k-1} \hat{\underline{\theta}}_{k-1/k-1} \quad (10)$$

$$\hat{\underline{\theta}}_{k/k} = \hat{\underline{\theta}}_{k/k-1} - g_k (z_k - h_k \hat{\underline{\theta}}_{k/k-1}) \quad (11)$$

where g_k are the filter gains.

The recursive equations for the filter gain and covariance matrixes are defined by:

$$\underline{g}_k = P_{k/k-1} \underline{h}_k^T (\underline{h}_k P_{k/k-1} \underline{h}_k^T + R_k)^{-1} \quad (12)$$

$$P_{k/k} = P_{k/k-1} - \underline{g}_k \underline{h}_k P_{k/k-1} \quad (13)$$

$$P_{k+1/k} = \Phi_{k+1,k} P_{k/k} \Phi_{k+1,k}^T + Q \quad (14)$$

Q is a measure of random excitation in the process and for a stationary emitter is omitted.

No estimate of the apriori value of $\underline{\theta}$ is available. Therefore, the filter is initialized with the first measurement in the following fashion:

$$\hat{\theta}_1 = z_1 \quad (15)$$

$$\hat{\dot{\theta}}_1 = (v/R) \sin(z_1) \quad (16)$$

In equation (16), v is the velocity of the observer, R the distance to the target, and z_1 the measured bearing angle. The only unknown in this equation is R , but this can be approximated. A value of 150 nmi is assumed in the computer simulation.

During simulation it was found that a simple $\Phi_{k,k-1}$ matrix of the form $\Phi_{k,k-1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ does not adequately represent the system. This can be seen in Figs. 2 and 3 where the mean errors of bearing angle and angular rate are shown using the simple and an augmented $\Phi_{k,k-1}$ matrix in a Monte Carlo simulation. The augmentation was found

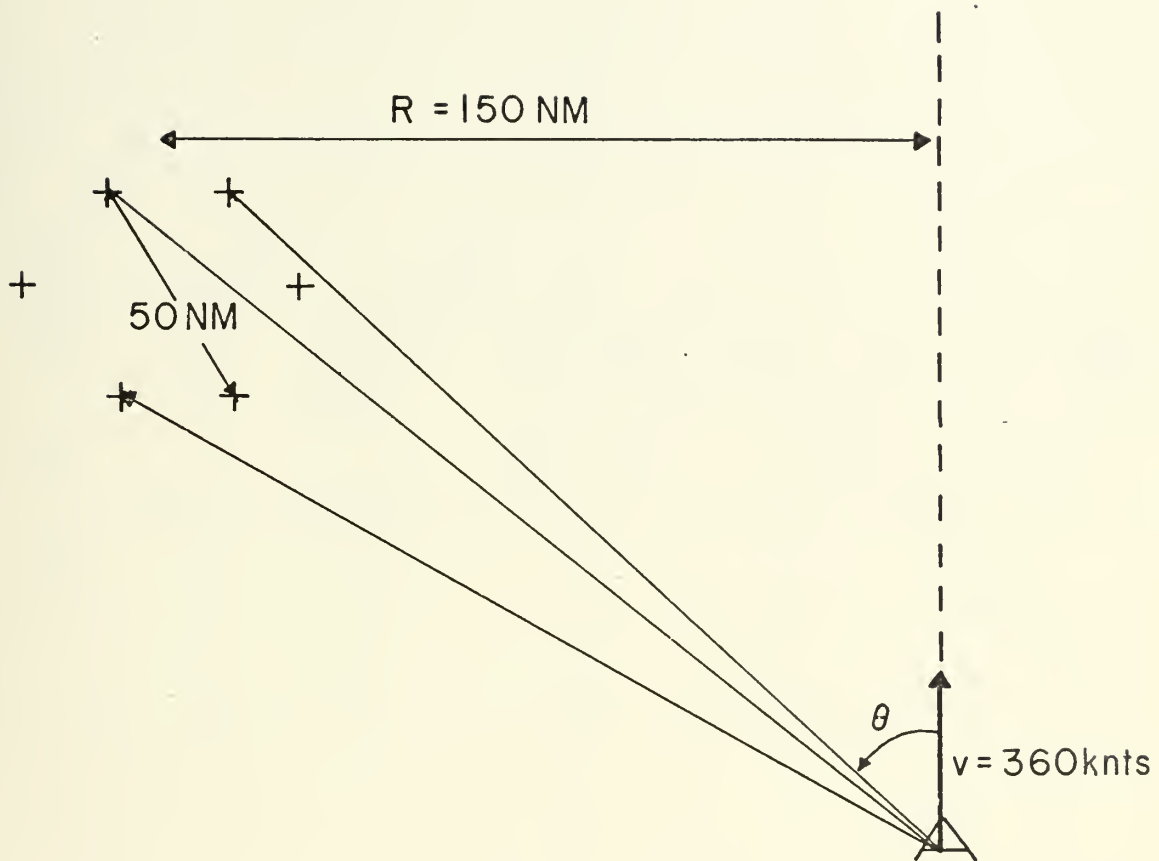


FIG. 1. SIMULATION SCENARIO AT FIRST OBSERVATION POINT WITH THREE OF SIX TARGETS RANDOMLY EMITTING AT AN AVERAGE RANGE OF 150 NMI.

INITIALIZATION EQUATION: $\dot{\theta} = (v/R) \sin^2(\theta)$

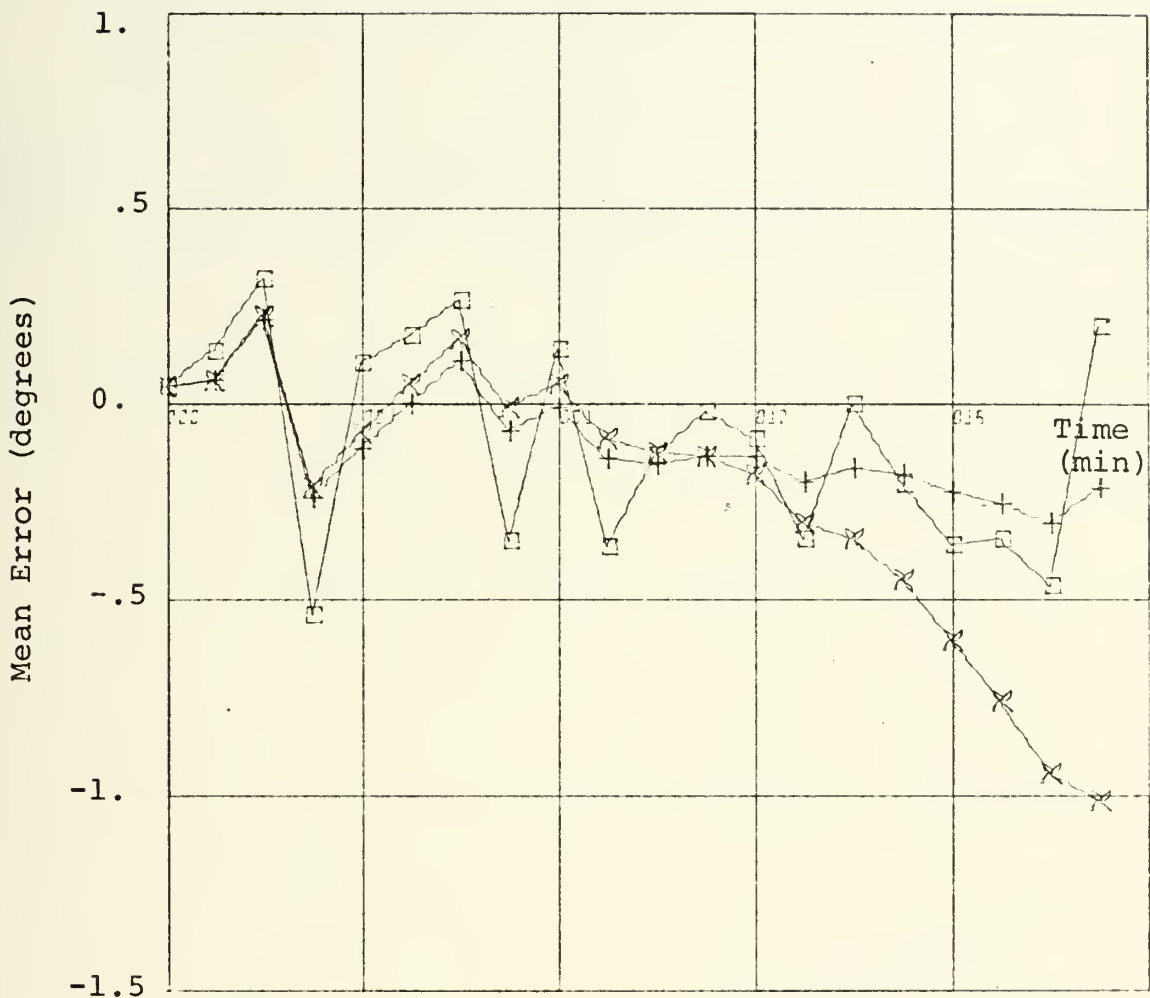


FIG. 2. MEAN ERROR OF BEARING ANGLE OBTAINED FROM A MONTE CARLO SIMULATION ON THE KALMAN FILTER WITH A SINGLE TARGET AND PERFECT CORRELATION FOR A TOTAL OF 20 STATION POINTS.

- MEAN ERROR OF THE MEASUREMENTS
- + MEAN ERROR OF ANGLE ESTIMATES USING A NONLINEAR Φ MATRIX IN THE KALMAN FILTER PREDICTION EQUATION
- X MEAN ERROR OF ANGLE ESTIMATE USING A LINEAR Φ MATRIX IN THE KALMAN FILTER PREDICTION EQUATION

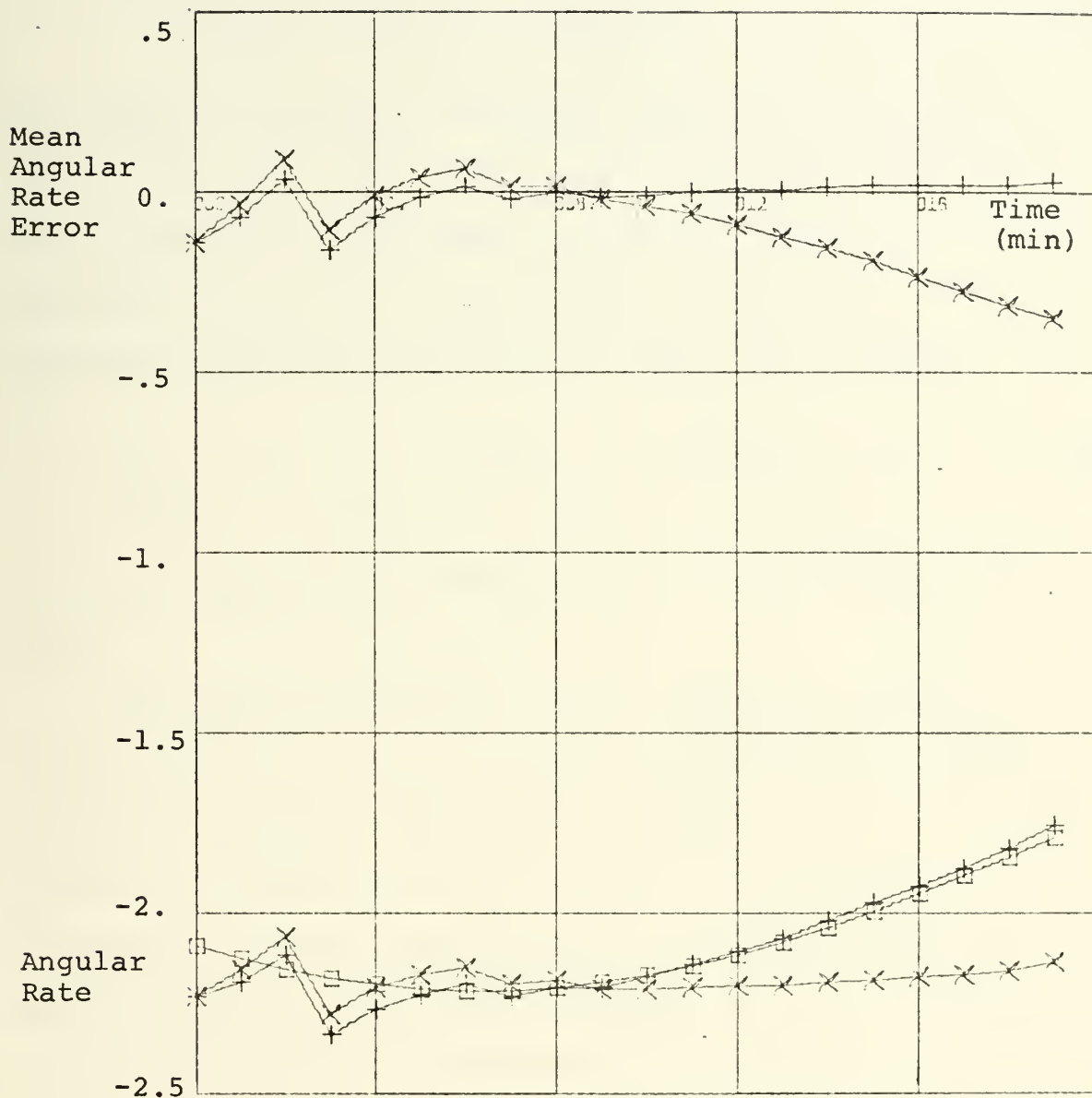


FIG. 3. MEAN ESTIMATE OF ANGULAR RATE AND MEAN RATE ERROR

- TRUE ANGULAR RATE
- + MEAN ESTIMATE OF ANGULAR RATE AND MEAN RATE ERROR
OBTAINED WHEN USING NONLINEAR Φ MATRIX
- x MEAN ESTIMATE OF ANGULAR RATE AND MEAN RATE ERROR
OBTAINED WHEN USING LINEAR Φ MATRIX

by introducing an additional difference equation:

$$\hat{\ddot{\theta}}_{k/k-1} = \hat{\ddot{\theta}}_{k-1/k-1} + \ddot{\theta}_{k/k-1}^T \quad (17)$$

The addition of another variable, the second derivative of the bearing angle, does not greatly complicate the computation; because for stationary targets (see Fig. 1), $\hat{\ddot{\theta}}$ can be expressed in terms of already defined variables. The augmented recursion equation for $\hat{\ddot{\theta}}_{k/k-1}$ then becomes:

$$\hat{\ddot{\theta}}_{k/k-1} = \hat{\ddot{\theta}}_{k-1/k-1} (1 + 2 * \hat{\ddot{\theta}}_{k-1/k-1} * \cot(\hat{\theta}_{k/k-1})^T) \quad (18)$$

This did improve the filtering results and eliminated the bias.

The easiest and fastest way to compute an apriori target location using the results obtained by the Kalman filter is to compute the target location based on the first observation made and on the estimate of bearing angle obtained at a later observation site, after a number of observations have been made. However, the first observation is an unfiltered measurement, and therefore may have quite a large error which will degrade the apriori target position seriously. To overcome this difficulty, the measurements are not only used in the Kalman filter to compute a current estimate, but also to obtain a smoothed estimate of the first observation. This smoothing is done by a technique proposed by H. E. Rauch[2]. The smoothed

estimate of the initial observation after k measurements are taken is given by:

$$\hat{\theta}_{1/k} = \hat{\theta}_{1/k-1} + D_{1/k} g_k (z_k - h_k \hat{\theta}_{k/k-1}) \quad (19)$$

$$D_{1/k} = D_{1/k-1} P_{k-1/k-1} \phi_{k,k-1}^T P_{k/k-1}^{-1} \quad (20)$$

Equation (20) shows that if the simple ϕ matrix is used, $D_{1/k}$ is data independent and may be precomputed and stored. For this case, and using the initial values given in Appendix A,

$$D_{1/k} = \begin{bmatrix} 1 & -(k-1) \\ 0 & 1 \end{bmatrix}$$

and since only a smoothed value of the bearing angle and not the angular rate is required, only a single scalar equation is needed to obtain a smoothed estimate of the initial bearing angle.

This leads to the following set of recursive filtering and smoothing equations in scalar form:

$$\begin{aligned} \hat{\theta}_{k/k-1} &= \hat{\theta}_{k-1/k-1} + T \hat{\dot{\theta}}_{k-1/k-1} \\ \hat{\dot{\theta}}_{k/k-1} &= \hat{\dot{\theta}}_{k-1/k-1} (1 + 2T \hat{\theta}_{k-1/k-1} \cot(\hat{\theta}_{k/k-1})) \\ \hat{\theta}_{k/k} &= \hat{\theta}_{k/k-1} + g_k(1) (z_k - \hat{\theta}_{k/k-1}) \\ \hat{\dot{\theta}}_{k/k} &= \hat{\dot{\theta}}_{k/k-1} + g_k(2) (z_k - \hat{\theta}_{k/k-1}) \\ \hat{\theta}_{1/k} &= \hat{\theta}_{1/k-1} + g_k(1) (z_k - \hat{\theta}_{k/k-1}) - (k-1) g_k(2) (z_k - \hat{\theta}_{k/k-1}) \end{aligned} \quad (21)$$

C. PARAMETER CORRELATION

The purpose of correlation is to assign observations to already established tracks if possible, or flag an observation as a new target if this observation does not correlate to previously established tracks. To each observation (or track) five parameters are associated:

Frequency

PRF (Pulse Repetition Frequency)

PW (Pulse Width)

Correlation counter

DOA (Direction of Arrival)

For each of these parameters a gate is selected. The size of the gate depends on the expected accuracy of the measurement and the confidence associated with the track parameter in question.

First, an observation is correlated to a track if the values of all five parameters are within their respective gate width. This may correlate one observation with several tracks, or several observations with the same track. To eliminate this multiple correlation, four rejection rules and one control rule are used:

1. A track which correlates with several observations rejects any observation held in common with another track if the common observation is the only observation correlating with the other track.
2. A track correlating with several observations, some of which are not held in common with other tracks, rejects observations held in common with other tracks.

3. When several observations correlate with one track, the closest observation is correlated to that track.
4. When several tracks correlate with one observation, the observation is correlated with the closest track.
5. If an observation is found uncorrelated, and at the same time there are one or more tracks uncorrelated to any observation, then this observation is correlated to the first uncorrelated track found within the gates.

The gates are chosen to be two sigma of the target parameter measurement noise plus two sigma of the confidence interval associated with the track parameter estimate error which is a function of the number of observations already correlated to that track.

Rule 5 was included, because rules 3 and 4 can reject too many of the initial correlations. If, for instance, two observations are correlated to the same two tracks, it is possible that one of these observations--say, the first--is closer to both tracks than the second observation. Then rule 3 rejects the second observation from both tracks and leaves the first correlated to both tracks. Rule 4 then selects the closest track as correlated to the first observation. This leaves unconnected the second observation and one track which was correlated initially.

D. TARGET LOCATION AND CORRELATION BY CLUSTER POINTS

The intent of this location method is to find the most probable target locations based on observations made at several stations, flag the observations leading to this location, and give a standard deviation to reflect the quality of the obtained observation. Fig. 4a shows four observation stations (Points I, II, III, IV) and possible bearings at these stations. A look at this Figure indicates quite clearly that points X, Y, and Z are not only the most probable, but also the only logical choices for target locations. To find these locations by means of a digital computer, one observation from station I is selected (Fig. 4b), and crossing points with all bearings from all other stations are calculated. Then, the mean and standard deviation of these locations is found. Since in the close neighborhood of point X, three points are located; the mean position will be relatively close to this point, but the far away points will make the standard deviation large. Now all points outside a 1.1σ interval from the mean are eliminated, and with the remaining points a new mean and standard deviation is calculated. This procedure will eliminate erroneous bearing lines and will finally produce a mean very close to the point X with only the appropriate bearings from the other stations contributing to it. Now if the computed standard deviation is small enough to be less than a location confidence limit, X is called a probable target location. The observations x_1, x_2, x_3, x_4 are

flagged as correlated. These observations are deleted from further computations. Then the next observation from station I is selected and crossings with the remaining bearing lines from all other stations are calculated (Fig. 4c). Here again, by computing a mean location and associated standard deviation, and rejecting distant crossings, the computed mean should converge to a location close to Y with a standard deviation smaller than the preassigned limit. Then for the last case, ideally only bearings leading to point Z are left.

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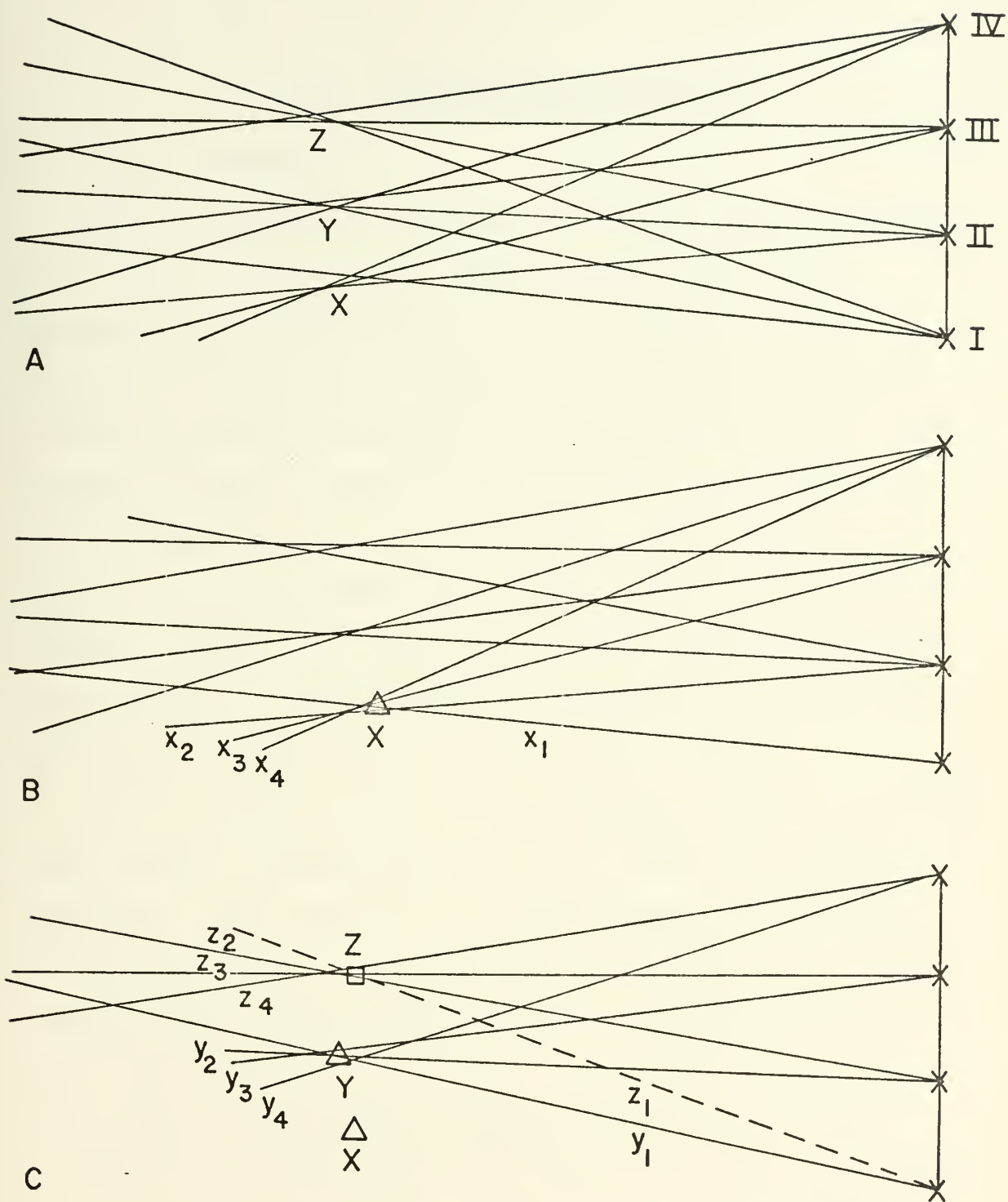


FIG. 4. STEPS FOR TARGET LOCATION BY "CLUSTER" POINTS

III. PRESENTATION OF RESULTS

A. TARGET LOCATION

For the computer simulation, two target configurations were taken. In the first, three targets were placed approximately 18 nmi apart on a line parallel to the heading of the observer aircraft. In the second, six targets were placed on a circle of approximately 50 nmi diameter, with the center of the circle approximately 125 nmi from the path of the observer. As a final aggravation, the probability of detection of any target was reduced to $1/2$; i.e., only three targets were emitting at any instant. A random sequence determined which three out of the six targets were observed at each station.

In the simulation the "cluster" method was first tested, using target configuration one; Fig. 5 shows the results obtained. Only two out of the three targets were found. The position errors were less than ten miles. The third target could be recovered by taking a second look at the still uncorrelated observations. To get an estimate on the spread of the computed target locations that could be expected, a Monte Carlo simulation was done. The result is shown in Fig. 6; the greatest error was approximately 50 nmi, and the center target was missed twice. The computed mean location from ten runs was on the far side for all three targets with an error of about 7 nmi.

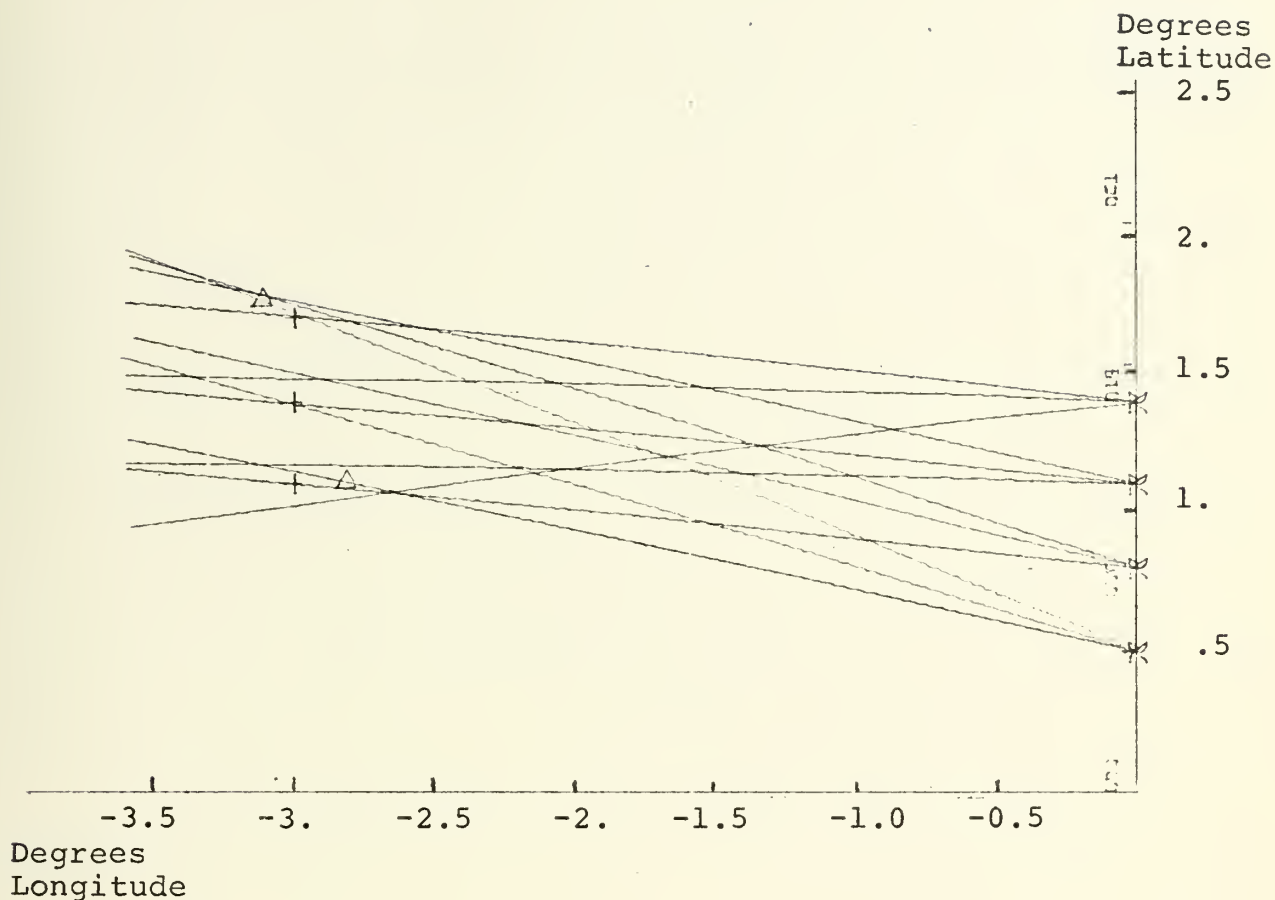


FIG. 5. TARGET LOCATION BY "CLUSTER" METHOD, SINGLE RUN. THREE TARGETS ARE APPROXIMATELY 18 NM APART; 4 OBSERVATION POINTS ARE ALSO APPROXIMATELY 18NM APART. THE DISTANCE BETWEEN OBSERVATIONS AND TARGETS IS APPROXIMATELY 150 NM. ALSO SHOWN ARE THE NOISY OBSERVATIONS MADE AT EACH STATION.

+ TRUE TARGET LOCATION

Δ CALCULATED "CLUSTER" TARGET LOCATION

X OBSERVATION POINT

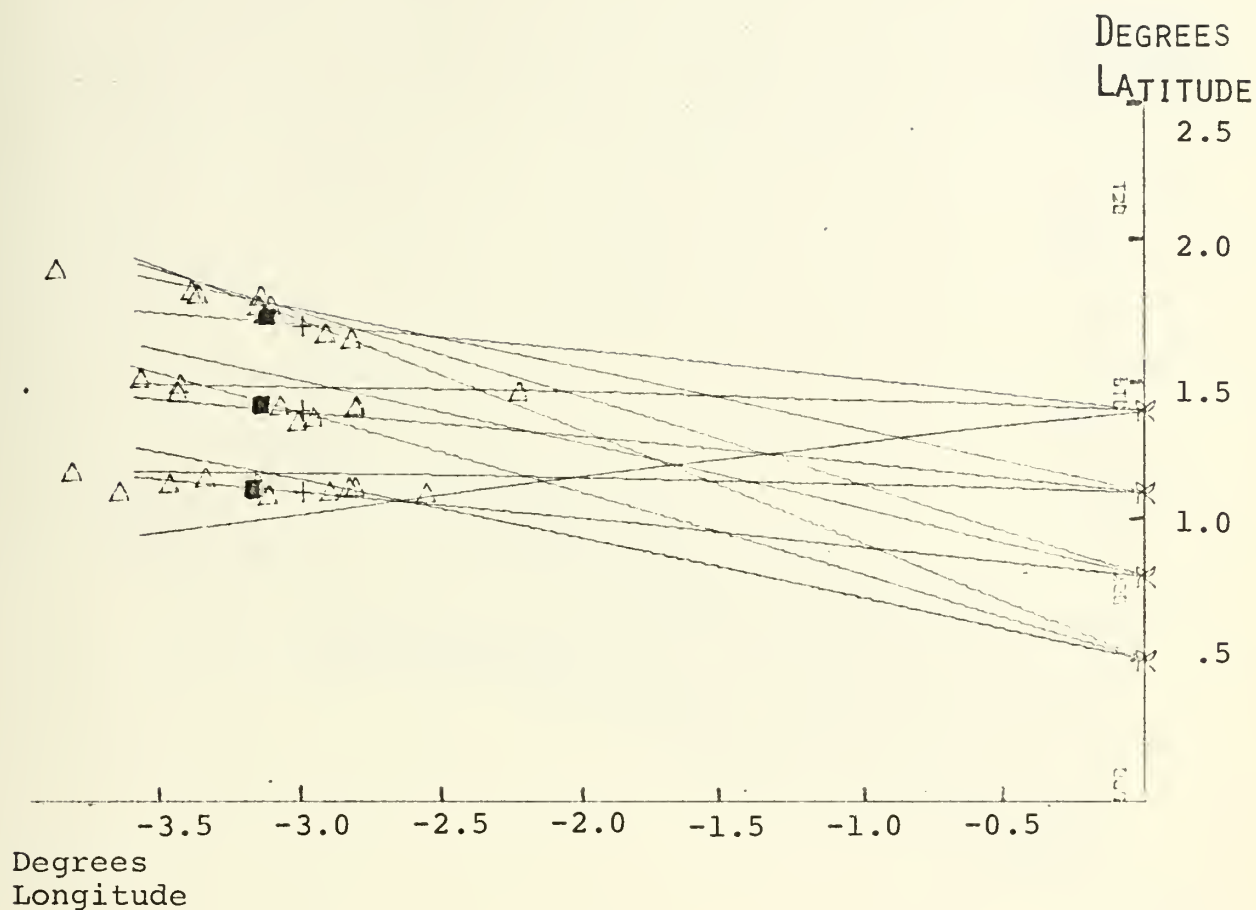


FIG. 6. TEN RUNS OF TARGET LOCATION BY "CLUSTER" METHOD

+ TRUE TARGET LOCATION

Δ COMPUTED TARGET LOCATION, INDIVIDUAL RUN

■ MEAN TARGET LOCATION FROM ALL RUNS

X OBSERVATION POINTS

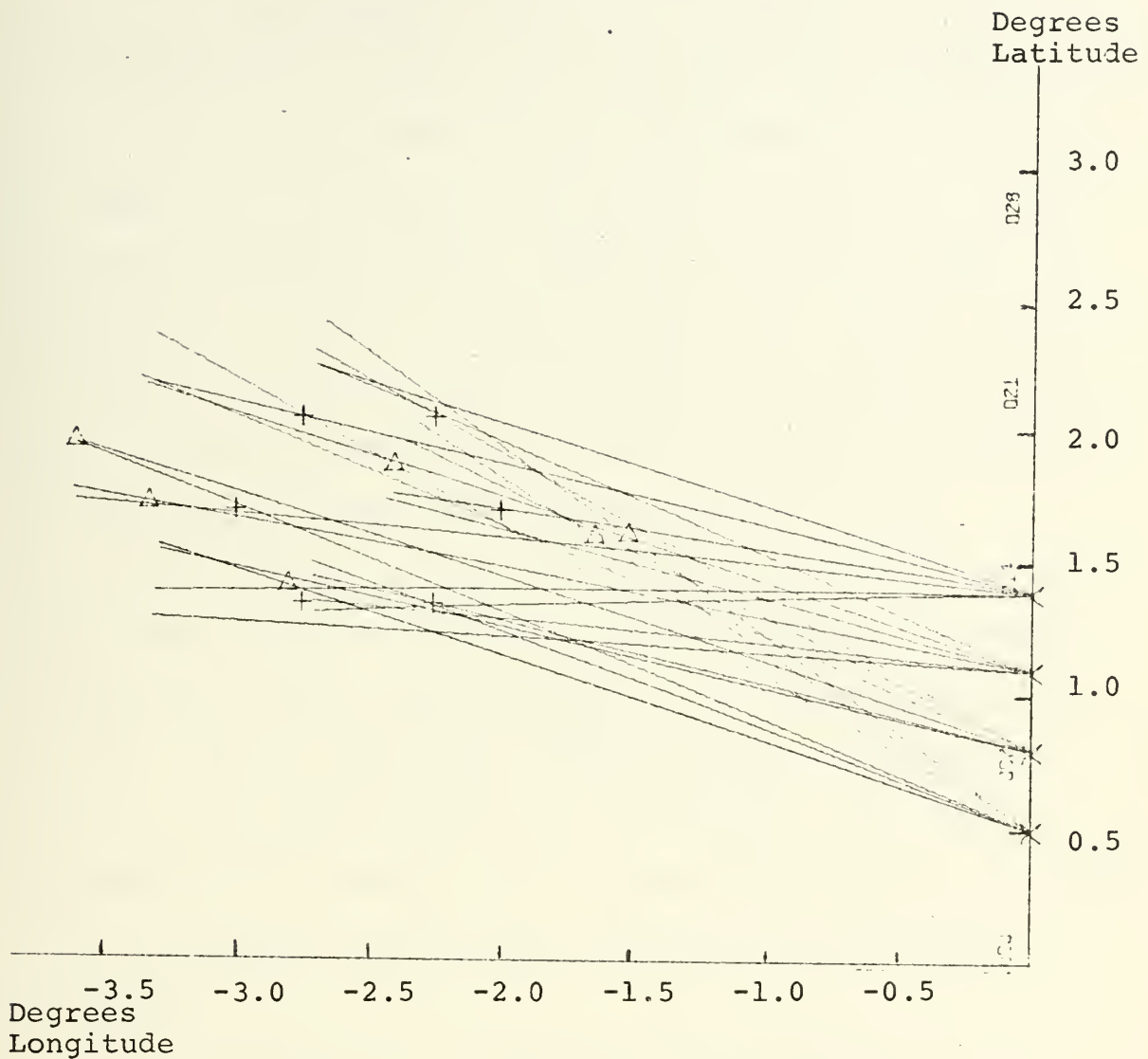


FIG. 7. TARGET LOCATIONS OBTAINED FROM "CLUSTER" METHOD. SIX TARGETS ARE PLACED ON A CIRCLE OF 25 NM RADIUS. THE CENTER OF THE CIRCLE IS 125 NM FROM THE BASE LINE OF OBSERVATION STATIONS.

- + TRUE TARGET LOCATION
- Δ COMPUTED TARGET LOCATION
- X LOCATION OF OBSERVATION POINTS

The results using the second target configuration show only one of the computed locations is reasonably close to the corresponding target, and that looking for cluster points is not very effective in a multi-emitter environment. The number of false locations becomes so high that the possibility of random clusters formed is too high to make the occurrence of clusters a good criterion for target location. Therefore, a new method was developed using a Kalman filter, and a correlation procedure to obtain refined estimates of the observation angles from the measurements taken. The result is shown in Fig. 8; all positions except one are closer than 10 nmi to the true target position. To allow a comparison between the two methods, the same distance from the first to the last observation point was chosen. In Fig. 9 the tracking distance is doubled; this gave a considerable improvement. The largest error now was 4.3 nmi with 3 positions closer than 2 nmi to the true target positions.

As a final test, the probability of detection was reduced to $1/2$. Fig. 10 shows the results obtained; given on the graph are the initial bearing line (smoothed) where a new track was initialized; the estimates of bearing lines at the last station; the target location based on only these two bearings (0); the location based on all estimates available (\square); the true target locations (+); and the observation points (X). The result reflect the missing observations. However, the computed target positions are

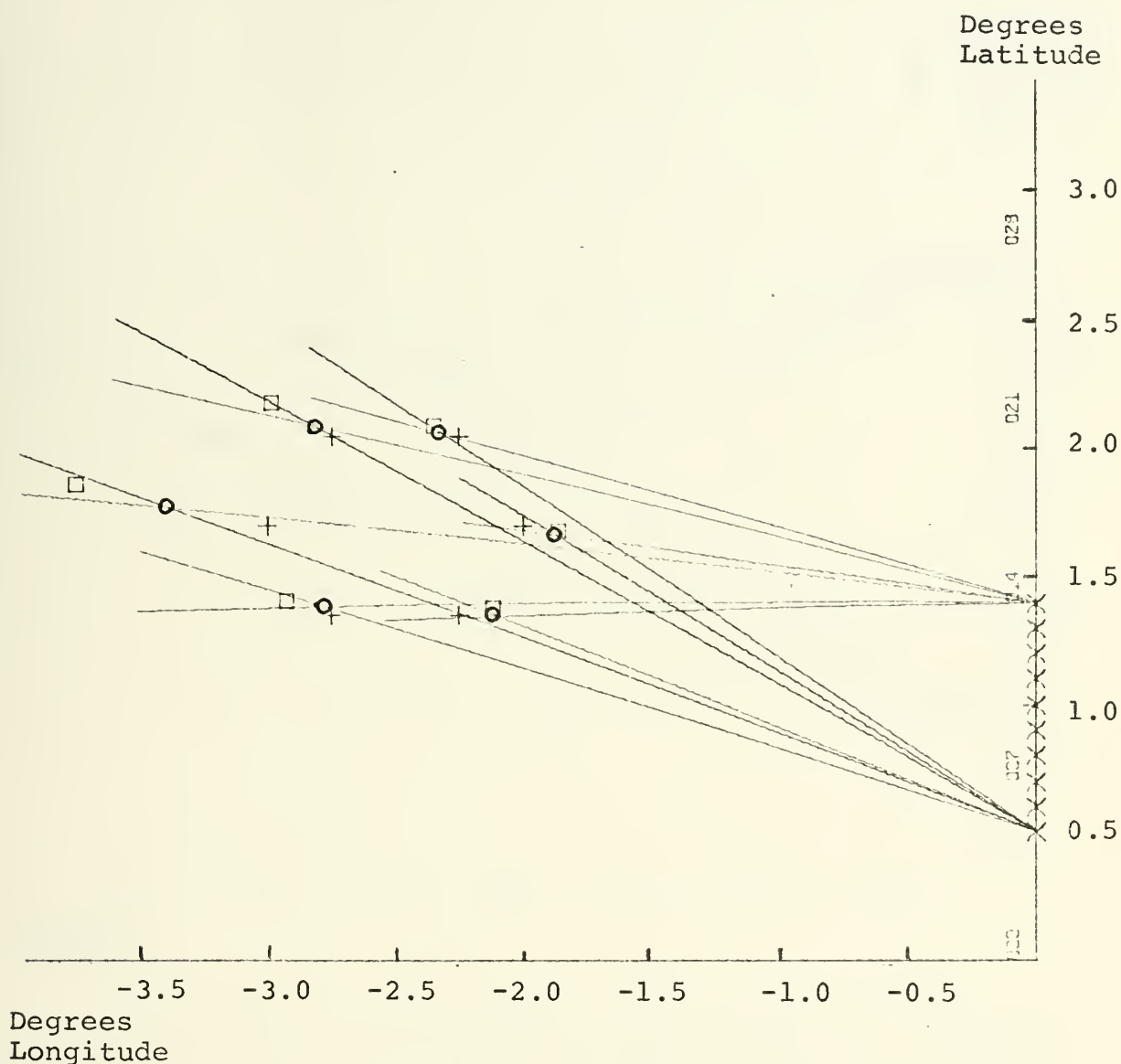


FIG. 8. TARGET LOCATIONS OBTAINED WHEN USING KALMAN FILTER ON OBSERVATION ANGLES. THE BEARING LINES SHOWN ARE THE SMOOTHED ESTIMATES OBTAINED AT THE FIRST AND LAST OBSERVATION POINTS.

+ TRUE TARGET LOCATION

□ MEAN TARGET LOCATION OBTAINED WHEN USING ALL BEARING ESTIMATES AVAILABLE, $\left(\frac{n(n-1)}{2}\right)$ LOCATIONS FOR EACH TARGET)

○ TARGET LOCATION USING BEARING ESTIMATE OF FIRST AND LAST OBSERVATION POINT ONLY

X OBSERVATION POINTS

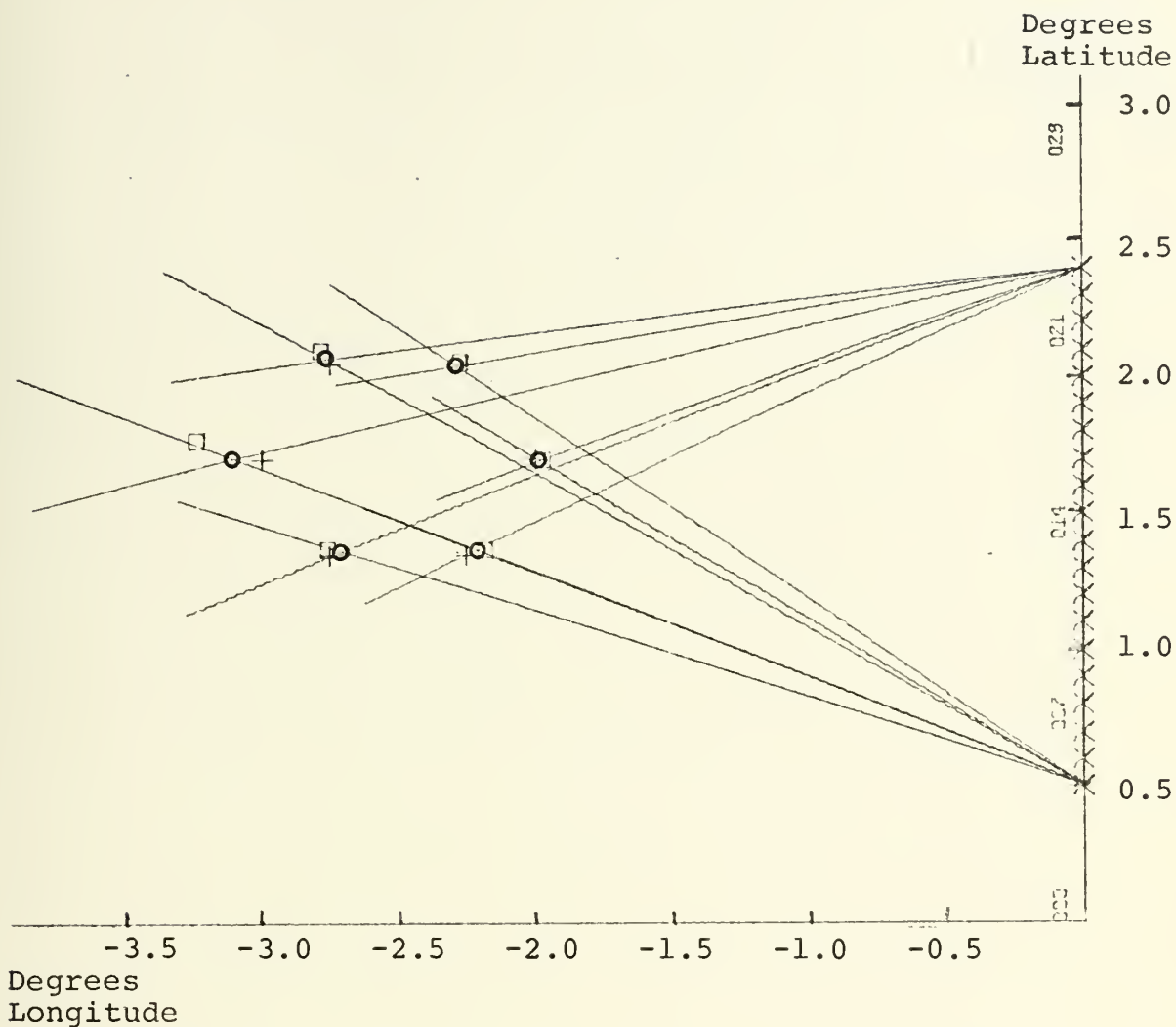


FIG. 9. TARGET LOCATION USING KALMAN FILTER WITH 20 OBSERVATION STATIONS. ALL TARGETS ARE OBSERVED AT ALL STATIONS.

+ TRUE TARGET LOCATION

□ MEAN TARGET LOCATION USING ALL AVAILABLE BEARING ESTIMATES

○ TARGET LOCATION USING FIRST AND LAST BEARING ESTIMATE ONLY

x OBSERVATION POINTS

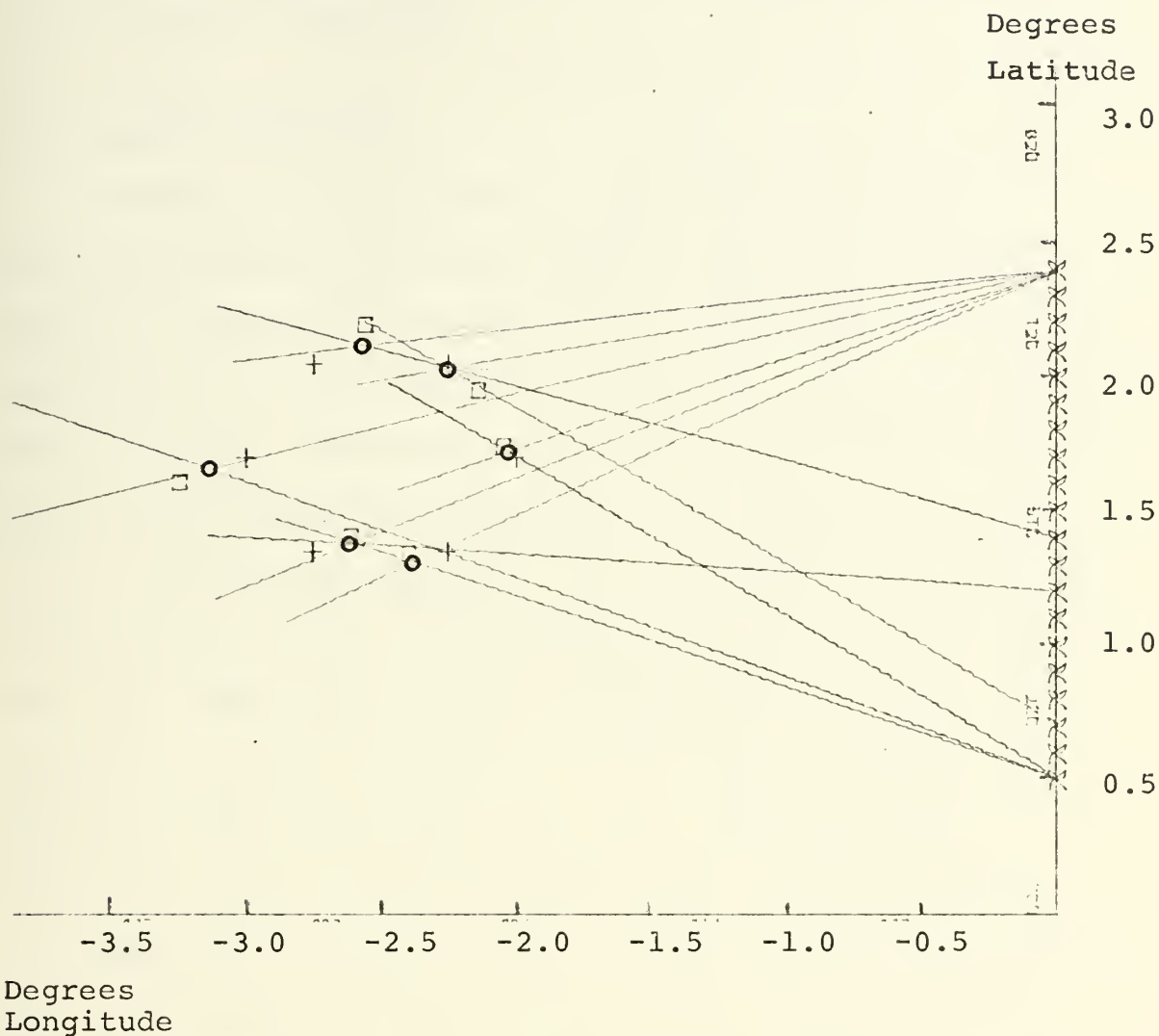


FIG. 10. TARGET LOCATION USING KALMAN FILTER WITH 20 OBSERVATION STATIONS. ONLY 3 TARGETS ARE OBSERVED AT ANY ONE STATION. THE GRAPH SHOWS THE ESTIMATE OF THE INITIAL OBSERVATION AND THE LAST OBSERVATION OF EACH TRACK.

+ TRUE TARGET LOCATION

□ MEAN TARGET LOCATION BASED ON ALL AVAILABLE ESTIMATES

○ TARGET LOCATION BASED ON FIRST AND LAST BEARING ESTIMATE ONLY

still close enough to the true positions to be useful as apriori target locations.

B. TARGET CORRELATION

A source of error arises in the target correlation. Two conflicting requirements make a compromise necessary. First, to reduce the possibility of correlating with wrong observations, it is desirable to select a small gate size, This, however, increases the possibility of missing observations which should be correlated with a track but happen to be noisier than expected. This is undesirable, because an uncorrelated observation will cause initialization of a new track very close to the already existing track. Now these two tracks will both be correlated to observations made from the same target, and these observations will be split by the two tracks--degrading the quality of both. So, false initialization of new tracks is even less desirable than allowing the possibility of a false correlation. The performance of the correlation method was evaluated for several test runs. Tables I and II show two typical cases. The numbering of tracks and observations corresponds to the numbering of the targets shown in Fig. 7.

The percentage of wrong correlations of 20% and 33.3% is quite high and degrades the filter performance seriously. Further studies should be done to find a more effective way of correlation. Even when noise-free observations were used, the noise introduced by the filter due to the approximation of the initial angular rate and the closeness

T R A C K S

O
B
S
E
R
V
A
T
I
O
N
S

	1	2	3	4	5	6
1	15		2	2	1	
2		16	4			
3	2	4	14			
4	2			16		2
5	1				18	1
6				2	1	17

TABLE I

CORRELATION RESULT FROM 20 OBSERVATION POINTS WITH ALL
6 TARGETS OBSERVED AT EACH STATION. ALL TRACKS ARE
INITIALIZED AT THE FIRST STATION.

120 OBSERVATIONS

24 WRONG CORRELATIONS ----> 20%

T R A C K S

O
B
S
E
R
V
A
T
I
O
N
S

	1	2	3	4	5	6
1	14 (1)					
2		7 (3)	1 (10)			
3	1	4	2			
4	2			6 (1)		
5				1	4	6 (1)
6				1	3 (8)	7

TABLE II

CORRELATION RESULT FROM 20 OBSERVATION POINTS WITH 3 TARGETS OBSERVED AT EACH STATION. THE NUMBER IN PARENTHESIS GIVES THE STATION AT WHICH THE TRACK WAS INITIALIZED. TRACK NUMBERS ARE ASSIGNED EQUAL TO THE TARGET NUMBER THAT HAS CONTRIBUTED THE MOST OBSERVATIONS TO THIS TRACK.

60 OBSERVATIONS

20 WRONG CORRELATIONS ---➔ 33.3%

of the targets considered was sufficient to cause correlation errors. This was particularly so in the low probability of detection case, when targets were emitting close together, and only one track was initialized. Then this track tends to pull in the observations from both targets, unless they are observed simultaneously at the same station.

C. STATISTICAL EVALUATION

Finally, a Monte Carlo simulation was performed using a single target (and ideal correlation) to verify that bias-free estimates are generated, and that the quality of the estimates are properly represented by the analytical covariances obtained. The evaluation is done for the Kalman filter, with and without the correction in the $\Phi(2,2)$ term. In both cases, the covariance sequences of the estimates of bearing angle and angular rate are reasonably close to the analytical values (Fig. 11, 12); but without the correction term, a bias is built up in the estimate of bearing angle and the estimate of the angular rate. With the correction term, no bias is present, and the mean error of the estimate approaches zero (Fig. 2,3).

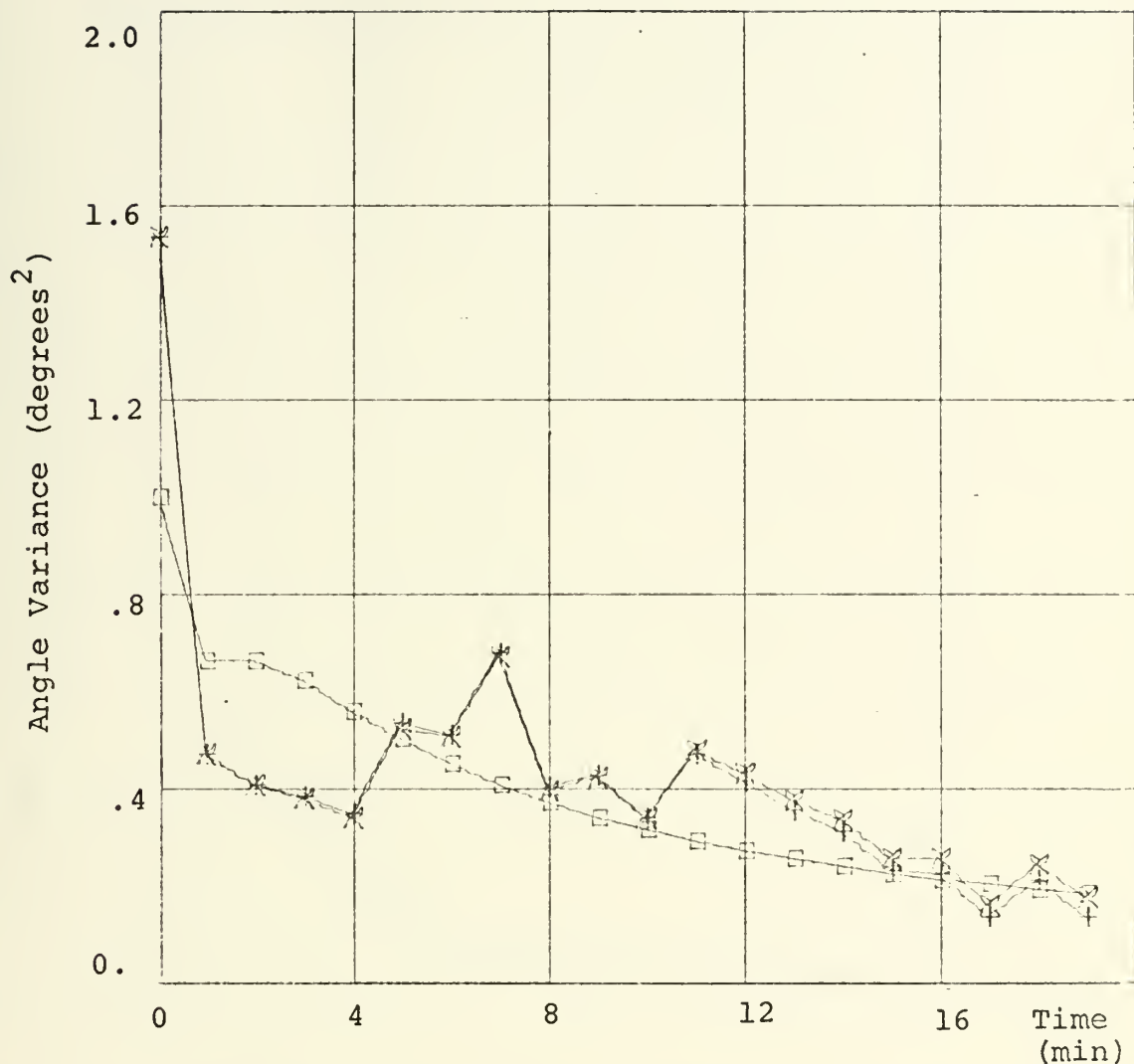


FIG. 11. VARIANCE OF BEARING ANGLE ESTIMATE OBTAINED FROM A MONTE CARLO SIMULATION ON THE KALMAN FILTER WITH A SINGLE TARGET AND PERFECT CORRELATION. FIRST OBSERVATION IS AT TIME ZERO.

□ ANALYTICAL VARIANCE OF THE ANGLE ESTIMATE

+ EXPERIMENTAL VARIANCE OF THE ANGLE ESTIMATE WHEN USING NONLINEAR Φ MATRIX IN KALMAN FILTER PREDICTION EQUATION

x EXPERIMENTAL VARIANCE OF THE ANGLE ESTIMATE WHEN USING LINEAR Φ MATRIX IN KALMAN FILTER PREDICTION EQUATION.

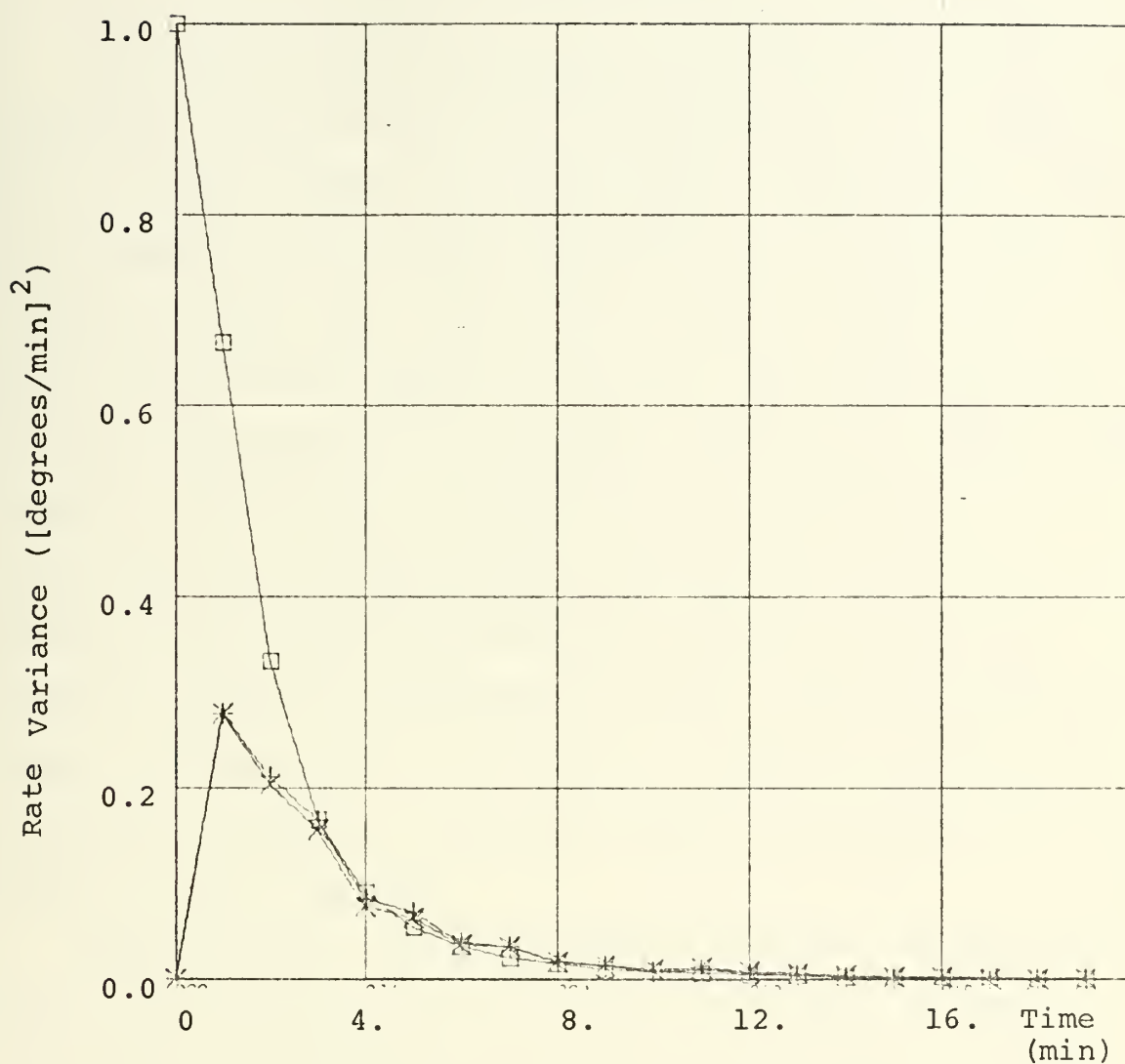


FIG. 12. VARIANCE OF ESTIMATE OF ANGULAR RATE OBTAINED FROM A MONTE CARLO SIMULATION ON THE KALMAN FILTER WITH A SINGLE TARGET AND PERFECT CORRELATION.

- ANALYTICAL VARIANCE OF THE RATE ESTIMATE
- + EXPERIMENTAL VARIANCE OF THE RATE ESTIMATE WHEN USING NONLINEAR Φ MATRIX
- x EXPERIMENTAL VARIANCE OF THE RATE ESTIMATE WHEN USING LINEAR Φ MATRIX

IV. CONCLUSIONS

The results from the computer simulation showed that a straight forward search for cluster points is not effective in a high-density, multiple-emitter environment. This is so because if m targets are emitting, and there are n observations, only $1/m$ of all possible locations are true locations--all others are wrong! This ratio of true to faulty locations can be reduced, and this was done in the computer simulation by first eliminating locations on the wrong side of the flight path, eliminating locations too distant to be considered meaningful, and by considering locations along selected bearings only. The high number of faulty locations still remaining is sufficient to generate erroneous clusters to which the process may converge and lead to false locations.

When a Kalman filter was used to process the measurements, the critical point was the correlation procedure which selects the measurements and assigns them to a track. If this selection was done perfectly, the filter would have only the proper data as its input and would produce an optimal estimate of the systems state. As correlation errors occur, wrong data is fed into the filter and performance deteriorates. This indicates that the overall system is only as good as the correlation. For this reason, the correlation procedure is quite elaborate; adaptive gating was introduced and five rules are used to handle the

multiple correlation problem as efficiently as possible. However, it must be pointed out that the two requirements for a good correlation method--correlation and discrimination--in a sense contradict each other and cannot both be maximized at the same time. In the initial correlation, a greater weight is put on the correlation; and the 4 rejection rules provide discrimination, but errors do occur during this rejection process, and the high error percentage obtained suggests that a better correlation method could be found which then, in turn, would improve the filter performance and the quality of locations obtained.

APPENDIX A

KALMAN FILTER BLOCK DIAGRAM AND INITIALIZATION

Recursive gain and covariance equations for filtering:

$$\underline{g}_K = \underline{P}_{K/K-1} \underline{h}_K^T [\underline{h}_K \underline{P}_{K/K-1} \underline{h}_K^T + R_K]^{-1}$$

$$\underline{P}_{K/K} = \underline{P}_{K/K-1} - \underline{g}_K \underline{h}_K \underline{P}_{K/K-1}$$

$$\underline{P}_{K+1/K} = \underline{\phi}_{K+1,K} \underline{P}_{K/K} \underline{\phi}_{K+1,K}^T + Q_K$$

Initial values:

$$\underline{P}_{1/0} = \begin{bmatrix} 10^4 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{\phi}_{K,K-1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$\underline{h}_K = [1 \quad 0]$$

$$R_K = 1$$

$$Q_K = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

for stationary targets.

Recursive gain and covariance equations for smoothing:

$$D_{1/K} = D_{1/K-1} P_{K-1/K-1} \Phi_{K,K-1}^T P_{K/K-1}^{-1}$$

$$P_{1/K} = P_{1/K-1} - D_{1/K} g_{K-K}^h P_{K/K-1} D_{1/K}^T$$

Initial values:

$$D_{1/1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_{1/0} = \begin{bmatrix} 10^4 & 0 \\ 0 & 1 \end{bmatrix}$$

Initial state values:

$$\hat{x}_{1/1} = \begin{bmatrix} z_1 \\ \frac{v}{R} \sin z_1 \end{bmatrix}$$

v = velocity of aircraft

R = 150 NM assumed initial distance to target

Note: In the Kalman filter equation a non-linear Φ matrix is used; when calculating gains and covariances, the linear approximation of the Φ matrix is used.

Kalman filter gain and covariance equations in scalar form.

$$g_K(1) = P_{K/K-1}(1,1) / [P_{K/K-1}(1,1) + R]$$

$$g_K(2) = P_{K/K-1}(1,2) / [P_{K/K-1}(1,1) + R]$$

$$P_{K+1/K}(1,1) = g_K(1)R + g_K(2)[2R - P_{K/K-1}(1,2)] + P_{K/K-1}(2,2)$$

$$P_{K+1/K}(1,2) = g_K(2)[R - P_{K/K-1}(1,2)] + P_{K/K-1}(2,2)$$

$$P_{K+1/K}(2,2) = P_{K/K-1}(2,2) - P_{K/K-1}(1,2)g_K(2)$$

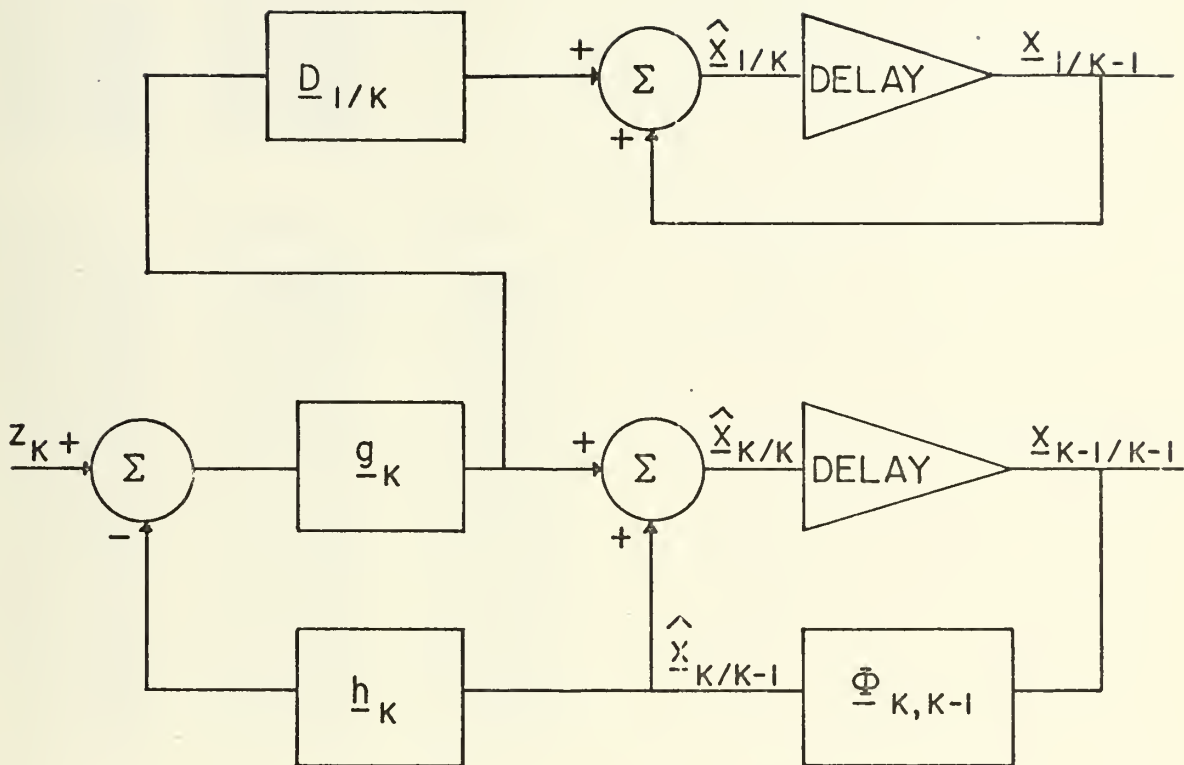


FIG. 13. BLOCK DIAGRAM OF KALMAN FILTER
AND SMOOTHING SOLUTION

COMPUTER RESULTS FOR SIX TARGETS AND 50% DETECTION

TARLAT	TARLON
30.70000	31.00000
31.04999	30.75000
31.04999	30.25000
30.70000	30.00000
30.34999	30.25000
30.34999	30.75000

STALAT	STALON	STDEV
29.50000	33.00000	1.00000
29.59999	33.00000	1.00000
29.70000	33.00000	1.00000
29.79999	33.00000	1.00000
29.89999	33.00000	1.00000
30.00000	33.00000	1.00000
30.09999	33.00000	1.00000
30.20000	33.00000	1.00000
30.29999	33.00000	1.00000
30.39999	33.00000	1.00000
30.50000	33.00000	1.00000
30.59999	33.00000	1.00000
30.70000	33.00000	1.00000
30.79999	33.00000	1.00000
30.89999	33.00000	1.00000
31.00000	33.00000	1.00000
31.09999	33.00000	1.00000
31.20000	33.00000	1.00000
31.29999	33.00000	1.00000
31.39999	33.00000	1.00000

BEARING AND DISTANCE FROM STATION 1 TO TARGET INDICATED

1	305.23	126.36
2	309.14	149.15
3	303.82	170.16
4	295.56	171.57
5	290.31	151.83
6	294.11	127.65

BEARING AND DISTANCE FROM STATION 2 TO TARGET INDICATED

1	302.95	122.99
2	307.31	145.43
3	302.11	166.89
4	293.73	169.07
5	288.16	149.85
6	291.60	125.31

BEARING AND DISTANCE FROM STATION 3 TO TARGET INDICATED

1	300.55	119.83
2	305.38	141.87
3	300.33	163.78
4	291.84	166.74
5	285.95	148.09
6	289.01	123.23

BEARING AND DISTANCE FROM STATION 4 TO TARGET INDICATED

1	298.02	116.89
2	303.36	138.49
3	298.49	160.84
4	289.90	164.60
5	283.70	146.55
6	286.33	121.41

BEARING AND DISTANCE FROM STATION 5 TO TARGET INDICATED

1	295.36	114.20
2	301.23	135.27
3	296.57	158.06
4	287.92	162.66
5	281.40	145.25
6	283.58	119.85

BEARING AND DISTANCE FROM STATION 6 TO TARGET INDICATED

1	292.58	111.76
2	299.01	132.27
3	294.60	155.48
4	285.88	160.91
5	279.06	144.18
6	280.76	118.59

BEARING AND DISTANCE FROM STATION 7 TO TARGET INDICATED

1	289.68	109.60
2	296.69	129.47
3	292.56	153.07
4	283.81	159.38
5	276.70	143.37
6	277.89	117.63

BEARING AND DISTANCE FROM STATION 8 TO TARGET INDICATED

1	286.68	107.73
2	294.27	126.88
3	290.45	150.87
4	281.70	158.06
5	274.31	142.78
6	274.98	116.94

BEARING AND DISTANCE FROM STATION 9 TO TARGET INDICATED

1	283.57	106.16
2	291.75	124.54
3	288.29	148.88
4	279.56	156.95
5	271.90	142.46
6	272.04	116.58

BEARING AND DISTANCE FROM STATION 10 TO TARGET INDICATED

1	280.39	104.92
2	289.15	122.44
3	286.07	147.11
4	277.38	156.06
5	269.49	142.39
6	269.09	116.52

BEARING AND DISTANCE FROM STATION 11 TO TARGET INDICATED

1	277.14	104.00
2	286.45	120.60
3	283.80	145.56
4	275.19	155.40
5	267.08	142.57
6	266.15	116.77

BEARING AND DISTANCE FROM STATION 12 TO TARGET INDICATED

1	273.84	103.42
2	283.68	119.04
3	281.49	144.25
4	272.98	154.97
5	264.68	143.00
6	263.23	117.33

BEARING AND DISTANCE FROM STATION 13 TO TARGET INDICATED

1	270.51	103.20
2	280.85	117.76
3	279.14	143.17
4	270.77	154.78
5	262.29	143.68
6	260.34	118.18

BEARING AND DISTANCE FROM STATION 14 TO TARGET INDICATED

1	267.18	103.32
2	277.96	116.79
3	276.75	142.35
4	268.55	154.82
5	259.94	144.61
6	257.50	119.35

BEARING AND DISTANCE FROM STATION 15 TO TARGET INDICATED

1	263.87	103.79
2	275.02	116.11
3	274.35	141.76
4	266.33	155.09
5	257.62	145.78
6	254.72	120.78

BEARING AND DISTANCE FROM STATION 16 TO TARGET INDICATED

1	260.60	104.60
2	272.06	115.74
3	271.92	141.43
4	264.13	155.58
5	255.34	147.17
6	252.01	122.49

BEARING AND DISTANCE FROM STATION 17 TO TARGET INDICATED

1	257.40	105.75
2	269.09	115.68
3	269.49	141.36
4	261.94	156.32
5	253.10	148.81
6	249.39	124.48

BEARING AND DISTANCE FROM STATION 18 TO TARGET INDICATED

1	254.27	107.21
2	266.13	115.93
3	267.07	141.54
4	259.78	157.26
5	250.92	150.66
6	246.85	126.72

BEARING AND DISTANCE FROM STATION 19 TO TARGET INDICATED

1	251.23	108.99
2	263.19	116.49
3	264.65	141.97
4	257.64	158.44
5	248.80	152.73
6	244.40	129.19

BEARING AND DISTANCE FROM STATION 20 TO TARGET INDICATED

1	248.30	111.06
2	260.28	117.35
3	262.25	142.66
4	255.54	159.83
5	246.73	155.00
6	242.05	131.90

ESTIMATES AT STATION 1

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	305.2349	306.4121	-1.8445	306.4121
4	295.5591	295.1960	-2.0739	295.1960
5	290.3093	290.5386	-2.1463	290.5386

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	304.567
9999.500	199.200	3.500	10.000	293.122
9999.500	199.200	3.500	10.000	288.392

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	293.828
10000.000	200.000	3.200	10.000	290.897
10000.000	200.000	3.200	10.000	303.179

INITIAL CORRELATION MATRIX

	TRACKS								
OBSERVATIONS	1	4	5	0	0	0	0	0	0
4	0	1	0						
5	0	1	1						
3	1	0	0						

FINAL CORRELATION MATRIX

	TRACKS								
OBSERVATIONS	1	4	5	0	0	0	0	0	0
4	0	1	0						
5	0	0	1						
3	1	0	0						

ESTIMATES AT STATION 2

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	302.9536	303.6416	-2.3862	305.9487
4	293.7261	293.5923	-1.9044	295.4312
5	288.1587	290.0620	-1.3704	291.3738

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	301.255
9999.500	199.200	3.500	10.000	291.688
9999.500	199.200	3.500	10.000	288.691

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	307.480
10000.000	200.000	3.200	10.000	285.740
10000.000	200.000	3.200	10.000	290.489

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	0	0	0	0	0	0	0
2	0	0	0							
5	0	0	1							
6	0	1	1							

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	0	0	0	0	0	0	0
2	0	0	0							
5	0	0	1							
6	0	1	0							

ESTIMATES AT STATION 3

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	300.5466	301.2554	-2.5068	305.9487
4	291.8401	290.8882	-2.3523	295.4307
5	285.9539	286.7239	-2.3736	291.3733
2	305.3792	307.4797	-1.9072	307.4797

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	9.000	298.749
9999.500	199.200	3.500	10.000	288.536
9999.500	199.200	3.500	10.000	284.350
10000.000	200.000	3.200	10.000	305.573

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	284.413
10000.000	200.000	3.200	10.000	286.115
10000.000	200.000	3.200	10.000	298.096

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
6	0	1	1	0						
5	0	1	1	0						
1	1	0	0	0						

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
6	0	0	1	0						
5	0	1	0	0						
1	1	0	0	0						

ESTIMATES AT STATION 4

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	298.0159	298.3137	-2.8423	305.9485
4	289.9031	287.0227	-3.0166	295.7332
5	283.6997	284.3892	-2.4085	291.3655
2	303.3562	305.5725	-1.9980	307.4797

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	295.471
9999.500	199.200	3.500	10.000	284.006
9999.500	199.200	3.500	10.000	281.980
10000.000	200.000	3.200	9.000	303.574

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	292.960
10000.000	200.000	3.200	10.000	299.960
10000.000	200.000	3.200	10.000	282.852

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
1	1	0	0	0						
2	0	0	0	1						
6	0	1	1	0						

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
1	1	0	0	0						
2	0	0	0	1						
6	0	0	1	0						

ESTIMATES AT STATION 5

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	295.3552	293.9016	-3.5951	306.2622
4	287.9155	284.0059	-3.0959	295.7332
5	281.4001	282.4714	-2.2949	291.2229
2	301.2344	301.1648	-3.2866	306.2742

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	290.306
9999.500	199.200	3.500	9.000	280.910
9999.500	199.200	3.500	10.000	280.177
10000.000	200.000	3.200	10.000	297.878

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	277.730
10000.000	200.000	3.200	10.000	283.602
10000.000	200.000	3.200	10.000	292.841

INITIAL CORRELATION MATRIX

	TRACKS									
	1	4	5	2	0	0	0	0	0	0
OBSERVATIONS										
5	0	1	1	0						
4	0	1	1	0						
1	1	0	0	0						

FINAL CORRELATION MATRIX

	TRACKS									
	1	4	5	2	0	0	0	0	0	0
OBSERVATIONS										
5	0	0	1	0						
4	0	1	0	0						
1	1	0	0	0						

ESTIMATES AT STATION 6

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	292.5771	291.7351	-3.3142	305.8477
4	285.8843	282.4272	-2.6802	295.2927
5	279.0637	278.9421	-2.6546	291.6421
2	299.0105	297.8782	-3.4860	306.2742

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	288.421
9999.500	199.200	3.500	10.000	279.747
9999.500	199.200	3.500	10.000	276.287
10000.000	200.000	3.200	9.000	294.392

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	290.938
10000.000	200.000	3.200	10.000	283.122
10000.000	200.000	3.200	10.000	296.286

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
1	1	0	0	1						
4	0	1	0	0						
2	0	0	0	1						

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
1	1	0	0	0						
4	0	1	0	0						
2	0	0	0	1						

ESTIMATES AT STATION 7

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	289.6799	289.6907	-3.1110	305.4155
4	283.8108	281.4497	-2.2746	294.7136
5	276.6963	276.2874	-2.6817	291.6421
2	296.6895	295.6548	-3.0584	306.2739

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	286.580
9999.500	199.200	3.500	10.000	279.175
9999.500	199.200	3.500	9.000	273.605
10000.000	200.000	3.200	10.000	292.596

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	274.281
10000.000	200.000	3.200	10.000	275.103
10000.000	200.000	3.200	10.000	288.125

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
5	0	0	1	0						
6	0	0	1	0						
1	1	0	0	0						

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	0	0	0	0	0	0
5	0	0	1	0						
6	0	0	0	0						
1	1	0	0	0						

ESTIMATES AT STATION 8

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	286.6750	287.2795	-3.0562	305.1560
4	281.6995	279.1750	-2.3038	294.7136
5	274.3062	273.9114	-2.6289	291.5286
2	294.2708	292.5962	-3.1943	306.2739
6	274.9802	275.1030	-2.2990	275.1030

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	284.223
9999.500	199.200	3.500	9.000	276.871
9999.500	199.200	3.500	10.000	271.282
10000.000	200.000	3.200	9.000	289.402
10000.000	200.000	3.200	10.000	272.804

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	285.256
10000.000	200.000	3.200	10.000	290.960
10000.000	200.000	3.200	10.000	272.636

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	0	0	0	0	0
1	1	0	0	1	0					
2	0	0	0	1	0					
6	0	0	1	0	1					

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	0	0	0	0	0
1	1	0	0	0	0					
2	0	0	0	1	0					
6	0	0	0	0	1					

ESTIMATES AT STATION 9

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	283.5730	284.6465	-3.0573	304.9905
4	279.5559	276.8711	-2.3261	294.7136
5	271.9006	271.2822	-2.6343	291.5286
2	291.7529	290.3755	-2.9371	306.0789
6	272.0425	272.6921	-2.3635	275.0469

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	281.589
9999.500	199.200	3.500	8.000	274.545
9999.500	199.200	3.500	9.000	268.648
10000.000	200.000	3.200	10.000	287.438
10000.000	200.000	3.200	10.000	270.329

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	280.466
10000.000	200.000	3.200	10.000	289.489
10000.000	200.000	3.200	10.000	287.626

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	0	0	0	0	0
1	1	0	0	0	0					
2	0	0	0	1	0					
3	0	0	0	1	0					

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	0	0	0	0	0
1	1	0	0	0	0					
2	0	0	0	0	0					
3	0	0	0	1	0					

ESTIMATES AT STATION 10

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	280.3870	281.1697	-3.1954	305.1599
4	277.3848	274.5449	-2.3411	294.7136
5	269.4885	268.6477	-2.6286	291.5286
2	289.1460	287.5442	-2.9981	306.0481
6	269.0938	270.3286	-2.3646	275.0469
2	289.1460	289.4893	-2.2181	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	277.974
9999.500	199.200	3.500	7.000	272.204
9999.500	199.200	3.500	8.000	266.019
10000.000	200.000	3.200	10.000	284.546
10000.000	200.000	3.200	9.000	267.964
10000.000	200.000	3.200	10.000	287.271

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	265.908
10000.000	200.000	3.200	10.000	277.801
10000.000	200.000	3.200	10.000	267.521

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
5	0	0	1	0	1	0				
1	1	0	0	0	0	0				
6	0	0	1	0	1	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
5	0	0	1	0	0	0				
1	1	0	0	0	0	0				
6	0	0	0	0	1	0				

ESTIMATES AT STATION 11

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	277.1357	277.9148	-3.2543	305.1843
4	275.1921	272.2036	-2.3485	294.7136
5	267.0781	265.9734	-2.6207	291.5461
2	286.4541	284.5459	-3.0795	306.0481
6	266.1504	267.6685	-2.5043	275.0466
2	286.4541	287.2710	-2.2715	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	274.660
9999.500	199.200	3.500	6.000	269.855
9999.500	199.200	3.500	10.000	263.353
10000.000	200.000	3.200	9.000	281.466
10000.000	200.000	3.200	10.000	265.164
10000.000	200.000	3.200	9.000	284.999

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	265.505
10000.000	200.000	3.200	10.000	274.497
10000.000	200.000	3.200	10.000	280.377

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
5	0	0	1	0	1	0				
1	1	0	0	0	0	0				
3	0	0	0	1	0	1				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
5	0	0	0	0	1	0				
1	1	0	0	0	0	0				
3	0	0	0	1	0	0				

ESTIMATES AT STATION 12

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	273.8369	274.6086	-3.2929	305.2207
4	272.9841	269.8550	-2.3480	294.7136
5	264.6780	263.3525	-2.5927	291.5461
2	283.6846	280.9167	-3.2905	306.2344
6	263.2268	265.3770	-2.4014	275.0039
2	283.6846	284.9993	-2.3198	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	271.316
9999.500	199.200	3.500	5.000	267.507
9999.500	199.200	3.500	9.000	260.760
10000.000	200.000	3.200	10.000	277.626
10000.000	200.000	3.200	10.000	262.975
10000.000	200.000	3.200	8.000	282.679

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	278.386
10000.000	200.000	3.200	10.000	259.362
10000.000	200.000	3.200	10.000	270.416

INITIAL CCRRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
3	0	0	0	1	0	1				
6	0	0	1	0	1	0				
4	1	1	0	0	0	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
3	0	0	0	1	0	0				
6	0	0	1	0	0	0				
4	1	0	0	0	0	0				

ESTIMATES AT STATION 13

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	270.5105	271.0515	-3.3342	305.3345
4	270.7659	267.5068	-2.3396	294.7136
5	262.2949	260.2375	-2.6440	291.7573
2	280.8472	277.9705	-3.2648	306.1067
6	260.3367	262.9753	-2.3766	275.0039
2	280.8472	282.6794	-2.3621	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	267.717
9999.500	199.200	3.500	4.000	265.167
9999.500	199.200	3.500	10.000	257.594
10000.000	200.000	3.200	10.000	274.706
10000.000	200.000	3.200	9.000	260.599
10000.000	200.000	3.200	7.000	280.317

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	266.405
10000.000	200.000	3.200	10.000	276.800
10000.000	200.000	3.200	10.000	257.448

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	1	1	0	0	0	0				
3	0	0	0	1	0	1				
6	0	0	1	0	1	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	0	1	0	0	0	0				
3	0	0	0	1	0	0				
6	0	0	1	0	0	0				

ESTIMATES AT STATION 14

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	267.1829	267.7173	-3.3187	305.3345
4	268.5466	265.7283	-2.1972	294.5056
5	259.9407	257.5435	-2.5980	291.7778
2	277.9558	275.5640	-3.1306	305.7715
6	257.4980	260.5986	-2.3439	275.0039
2	277.9558	280.3174	-2.3975	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	9.000	264.398
9999.500	199.200	3.500	10.000	263.531
9999.500	199.200	3.500	10.000	254.945
10000.000	200.000	3.200	10.000	272.433
10000.000	200.000	3.200	8.000	258.255
10000.000	200.000	3.200	6.000	277.920

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	273.934
10000.000	200.000	3.200	10.000	267.236
10000.000	200.000	3.200	10.000	257.372

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
2	0	0	0	1	0	1				
4	1	0	0	0	0	0				
5	0	0	1	0	1	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
2	0	0	0	1	0	0				
4	1	0	0	0	0	0				
5	0	0	0	0	1	0				

ESTIMATES AT STATION 15

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	263.8740	265.1750	-3.1932	304.9941
4	266.3318	263.5310	-2.1781	294.5056
5	257.6194	254.9455	-2.5346	291.7778
2	275.0237	272.9934	-3.0500	305.5442
6	254.7192	257.7568	-2.4628	275.1479
2	275.0237	277.9197	-2.4254	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	261.982
9999.500	199.200	3.500	9.000	261.353
9999.500	199.200	3.500	9.000	252.411
10000.000	200.000	3.200	10.000	269.943
10000.000	200.000	3.200	10.000	255.294
10000.000	200.000	3.200	5.000	275.494

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	262.133
10000.000	200.000	3.200	10.000	256.917
10000.000	200.000	3.200	10.000	272.597

INITIAL CORRELATION MATRIX

OBSERVATIONS	TRACKS									
	1	4	5	2	6	2	0	0	0	0
1	1	1	0	0	0	0				
5	0	0	0	0	1	0				
2	0	0	0	1	0	1				

FINAL CORRELATION MATRIX

OBSERVATIONS	TRACKS									
	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	0	0				
5	0	0	0	0	1	0				
2	0	0	0	1	0	0				

ESTIMATES AT STATION 16

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	260.6050	262.0203	-3.1390	304.9768
4	264.1274	261.3528	-2.1529	294.5056
5	255.3379	252.4109	-2.4635	291.7778
2	272.0647	270.8521	-2.9119	305.1655
6	252.0108	256.1128	-2.1910	274.8694
2	272.0647	275.4941	-2.4452	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	258.881
9999.500	199.200	3.500	8.000	259.200
9999.500	199.200	3.500	8.000	249.947
10000.000	200.000	3.200	10.000	267.940
10000.000	200.000	3.200	10.000	253.922
10000.000	200.000	3.200	4.000	273.049

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	261.553
10000.000	200.000	3.200	10.000	251.953
10000.000	200.000	3.200	10.000	250.358

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
4	1	1	0	0	0	0				
5	0	0	1	0	0	1				
6	0	0	1	0	1	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
4	0	1	0	0	0	0				
5	0	0	0	0	1	0				
6	0	0	1	0	0	0				

ESTIMATES AT STATION 17

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	257.3982	258.8811	-3.0714	304.9768
4	261.9414	260.1643	-1.9332	294.1287
5	253.1047	250.0772	-2.3646	291.6860
2	269.0950	267.9402	-2.9012	305.1655
6	249.3858	253.0294	-2.3437	275.1997
2	269.0950	273.0488	-2.4563	289.4893

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	9.000	255.810
9999.500	199.200	3.500	10.000	258.231
9999.500	199.200	3.500	10.000	247.713
10000.000	200.000	3.200	9.000	265.039
10000.000	200.000	3.200	10.000	250.686
10000.000	200.000	3.200	3.000	270.593

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	255.230
10000.000	200.000	3.200	10.000	268.186
10000.000	200.000	3.200	10.000	260.174

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	1	1	0	0	0	0				
3	0	0	0	1	0	1				
4	0	1	0	0	0	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	0	0				
3	0	0	0	0	0	1				
4	0	1	0	0	0	0				

ESTIMATES AT STATION 18

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	254.2676	255.6700	-3.0017	305.0388
4	259.7776	258.9563	-1.7804	293.8345
5	250.9225	247.7126	-2.2846	291.6860
2	266.1299	265.0388	-2.8757	305.1655
6	246.8473	250.6858	-2.2765	275.1997
2	266.1299	268.9878	-3.2545	288.6863

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	252.668
9999.500	199.200	3.500	10.000	257.176
9999.500	199.200	3.500	9.000	245.428
10000.000	200.000	3.200	8.000	262.163
10000.000	200.000	3.200	9.000	248.409
10000.000	200.000	3.200	10.000	265.733

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	251.941
10000.000	200.000	3.200	10.000	263.135
10000.000	200.000	3.200	10.000	245.247

INITIAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	1	0				
2	0	0	0	1	0	1				
6	0	0	1	0	1	0				

FINAL CORRELATION MATRIX

	TRACKS									
OBSERVATIONS	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	0	0				
2	0	0	0	1	0	0				
6	0	0	1	0	0	0				

ESTIMATES AT STATION 19

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	251.2313	252.5027	-2.9186	305.1133
4	257.6436	257.1758	-1.7552	293.8345
5	248.7965	245.3749	-2.2080	291.7087
2	263.1853	262.4705	-2.7851	304.9487
6	244.4004	248.4093	-2.2049	275.1997
2	263.1853	265.7332	-3.2269	288.6863

ORIGINAL MATRIX X OF PREDICTED TRACKS

FREQU	PRF	PW	COUNT	DOA
9999.500	199.200	3.500	10.000	249.584
9999.500	199.200	3.500	9.000	255.421
9999.500	199.200	3.500	10.000	243.167
10000.000	200.000	3.200	10.000	259.685
10000.000	200.000	3.200	8.000	246.204
10000.000	200.000	3.200	9.000	262.506

INPUT OBSERVATIONS

FREQU	PRF	PW	COUNT	DOA
10000.000	200.000	3.200	10.000	247.938
10000.000	200.000	3.200	10.000	263.600
10000.000	200.000	3.200	10.000	240.869

INITIAL CORRELATION MATRIX

OBSERVATIONS	TRACKS									
	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	1	0				
3	0	0	0	0	0	1				
6	0	0	1	0	0	0				

FINAL CORRELATION MATRIX

OBSERVATIONS	TRACKS									
	1	4	5	2	6	2	0	0	0	0
1	1	0	0	0	0	0				
3	0	0	0	0	0	1				
6	0	0	1	0	0	0				

ESTIMATES AT STATION 20

TRACK	THT	TH K/K	TD K/K	TH 1/K
1	248.3002	249.2296	-2.8380	305.2737
4	255.5435	255.4206	-1.7272	293.8345
5	246.7299	242.5381	-2.1949	291.9839
2	260.2766	259.6853	-2.7359	304.9487
6	242.0506	246.2044	-2.1300	275.1997
2	260.2766	263.2356	-2.8191	288.6863

COMPUTER PROGRAM FOR APRIORI TARGET LOCATION AND CORRELATION

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C  LIST OF VARIABLES
C      THT(I,J)      TRUE BEARING ANGLE
C      THETA(I,J)    MEASURED BEARING ANGLE
C      TH1KK(I,J)    FILTERED ESTIMATE OF BEARING ANGLE
C      TD1KK(I,J)    FILTERED ESTIMATE OF ANGULAR RATE
C      TH1(J)        SMOOTHED ESTIMATE OF FIRST BEARING ANGLE
C      E1(I,J)       ERROR OF TH1KK(I,J)
C      NJ(I,J)       RANDOM SEQUENCE OF TARGETS DETECTED
C
C  LIST OF SUBSCRIPTS
C      I              1.GE.I.LE.NSTA      STATION INDEX
C      K              1.GE.K.LE.20        K.F. INDEX
C  NOTE:  I AND K WILL IN GENERAL BE THE SAME
C      J              1.GE.J.LE.NTAR      TARGET INDEX
C
C      REAL*8 ITL(12),LBL/' '
C      REAL LABEL/' '
C      COMMON /A/ NTR,NJ,TH1,TD1KK,NFRST /B/TLAT,TLOT,ITL,X,Y
C      DIMENSION THT(20,20),THETA(20,20),STEEST(20,20),XX(20)
C      1TD1KK(20,20),GK(20),GD(20),PKK(20),NFRST(20),NJ(20,20)
C      1E1(20,20),TH(20),TLA(20),TLO(20),SLA(20),SLO(20),X(20)
C      1TLAT(20),OBS(5,20),TRK(5,20),GATE(5),ADDGT(20),YY(20),
C      1TLOT(20),NK1(20),NTR(20),Y(20),TH1KK(20,20),TH1(20),
C      1IC(20),SLA1(20),SLO1(20)
C
C      DATA GK /1.,.6667,.6667,.625,.5637,.5045,.4532,.4099,
C      X.3734,.3425,.3161,.2934,.2737,.2563,.2411,.2275,.2153,
C      X.2044,.1945,.1855/
C      DATA GD/0.,.3332,.3333,.25,.1818,.1352,.1035,.08141,
C      X.06558,.0539,.0539,.0382,.0328,.02845,.0249,.022,
C      X.01956,.0175,.01576,.01427/
C      DATA PKK/1.414,1.414,1.29,1.13,1.007,.91,.833,.772,
C      X.724,.68,.645,.615,.5865,.564,.542,.524,.507,.497,.477
C      X.4645/
C      DATA PI/3.141593 /,RD/.0174533/,NK1/20*0/,R/3959./
C      DATA GATE/1.,1.,3.,10.,2./
C
C      IX=38159
C      TDM=2.292
C      READ (5,102) (ITL(I),I=1,12)
C      DO 1000 J=1,6
C 1000 READ (5,103) (OBS(I,J),TRK(I,J),I=1,4)
C      DO 1001 K=1,20
C 1001 READ (5,105) (NJ(K,J),J=1,3)
C
C      READ (5,105) (NTR(J),J=1,20)
C      CALL BEARG1 (THT,STEEST,NSTA,NO,SLA,SLO)
C
C  INITIALIZATION
C
C      I=1
C      WRITE (6,101)
C      WRITE (6,106) I
C      NO=3
C      DO 10 J=1,NC
C      J1=NJ(1,J)
C      NTR(J)=J1
C      NFRST(J)=1
C      CALL GAUSS(IX,STEEST(1,J),THT(1,J1),THETA(1,J))
C      TH1KK(1,J)=THETA(1,J)
C      TD1KK(1,J)=TDM*SIN(THETA(1,J)*RD)
C      TH1(J)=THETA(1,J)
C      E1(1,J)=TH1KK(1,J)-THT(1,J1)
C      WRITE (6,100)NTR(J),THT(1,J1),TH1KK(1,J),TD1KK(1,J),
C      &TH1(J)
C 10 CONTINUE

```



```

C
C   LINEAR KALMAN FILTER WITH PREDET. GAINSCCHEDULE
C
C   NT1=NO
C   DO 12 I=2,NSTA
C
C   DO 11 J=1,NT1
C
C   IF(J.GT.NO) GOTO 1
C   J1=NJ(I,J)
C   CALL GAUSS(IX,STEEST(I,J),THT(I,J1),THETA(I,J))
C   OBS(5,J)=THETA(I,J)
1  K=I-NK1(J)
C
C   COMPUTE PREDICTED VALUE FOR CORRELATION
C
C   TRK(5,J)=TH1KK(I-1,J)+TD1KK(I-1,J)
C   ADDGT(J)=PKK(K)*2.
11  CONTINUE
C   WRITE (6,101)
C   CALL CORREL (IC,NO,NT1,OBS,TRK,GATE,ADDGT,NK1,I)
C   WRITE (6,106) I
C   DO 12 J=1,NT1
C
C   COMPUTE ESTIMATE
C
C   K=I-NK1(J)
C   E=OBS(5,J)-TRK(5,J)
C   ET=0.
C   ED=0.
C   IF(IC(J).EQ.0) GOTO 110
C   ET=GK(K)*E
C   ED=GD(K)*E
110 CONTINUE
C   TH1KK(I,J)=TRK(5,J)+ET
C   EPS=2.*TD1KK(I-1,J)*RD/TAN(TH1KK(I,J)*RD)
C   TDKKM=TD1KK(I-1,J)*(1.+EPS)
C   TD1KK(I,J)=TDKKM+ED
C   NTRJ=NTR(J)
C   E1(I,J)=TH1KK(I,J)-THT(I,NTRJ)
C   TH1(J)=TH1(J)+ET-(K-1)*ED
C
C   WRITE RESULTS
C
C   WRITE(6,100)NTRJ,THT(I,NTRJ),TH1KK(I,J),TD1KK(I,J),
C   &TH1(J)
12  CONTINUE
C
C   COMPUTE APRIORI TARGET LOCATION
C
C   WRITE(6,104)
C   DO 121 I=1,NT1
C   IF(TRK(4,I).LT.0) GOTO 121
C   TH(1)=TH1(I)
C   NFI=NFRST(I)
C   SLA1(1)=SLA(NFI)
C   SLO1(1)=SLO(NFI)
C   NS=NSTA-NFI+1
C   DO 120 J=2,NS
C   K=NFI+J-1
C   TH(J)=TH1KK(K,I)
C   SLA1(J)=SLA(K)
C   SLO1(J)=SLO(K)
120 CONTINUE
C   CALL FIND (TLA(I),TLO(I),SIG,NS,TH,SLA1,SLO1)
C   NTRJ=NTR(I)
C   DELLA=TLA(I)-TLAT(NTRJ)
C   DELLO=TLO(I)-TLOT(NTRJ)
C   RNGERR=SQRT((DELLA*60.)**2+(DELLO*50.)**2)
C   WRITE (6,107) TLA(I),TLO(I),DELLA,DELLO,RNGERR
121 CONTINUE
C

```


C PLOT NOISY BEARING LINES

```

C
DO 1002 L=1,2
DO 1002 I=1,NT1
IF(TRK(4,I).LT.0) GOTO 1002
K=NFRST(I)
IF(L.EQ.2) K=NSTA
XX(1)=X(K)
YY(1)=Y(K)
THR=TH1(I)
IF(L.EQ.2) THR=TH1KK(NSTA,I)
IF(THR.GT.180) THR=360.-THR
CA=ABS(TLO(I)-SLO(K))*RD
SC=(90.-SLA(K))*RD
SB=(90.-TLA(I))*RD
SB1=ARCOS(COS(SB)*COS(SC)+SIN(SB)*SIN(SC)*COS(CA))
DO 1010 J=1,6
SB=SB1*J/5.
SA=ARCOS(COS(SB)*COS(SC)+SIN(SB)*SIN(SC)*COS(THR*RD))
CB=ARCOS((COS(SB)-COS(SA)*COS(SC))/(SIN(SA)*SIN(SC)))
IF(SLO(K).LT.TLO(I)) GOTO 1008
THA=CB-SLO(K)*RD
GO TO 1009
1008 THA=-CB-SLO(K)*RD
1009 CONTINUE
THA=-THA/RD
PH=90.-SA/RD
CALL SHIFT (R,THA,PH,XX(J+1),YY(J+1))
1010 CONTINUE
CALL DRAW (7,XX,YY,2,0,LABEL,ITL,.2,.2,5,5,2,2,9,10,0,
1002 CONTINUE

```

C C PLOT COMPUTED TARGET POSITION

```

C
K=0
DO 13 I=1,NT1
IF(TRK(4,I).LT.0) GOTO 13
K=K+1
CALL SHIFT (1.,TLO(I),TLA(I),XX(K),YY(K))
13 CONTINUE
CALL DRAW (K,XX,YY,3,3,LBL,ITL,.1,.1,5,5,2,2,9,10,0,L)
STOP
100 FORMAT (I17,4X,10F11.4)
101 FORMAT (1H1)
102 FORMAT (6A8)
103 FORMAT (8F10.5)
104 FORMAT (1H1,////14X,'TARLAT',4X,'TARLON',4X,'DEL.LAT',
&3X,'DEL.LON',3X,'RNG.ERR',/)
105 FORMAT (40I2)
106 FORMAT (////14X,'ESTIMATES AT STATION',I3,////14X,'TRACK'
&,6X,'THT',8X,'TH K/K',5X,'TD K/K',5X,'TH 1/K',/)
107 FORMAT (11X,8F10.5)
END

```

SUBROUTINE SHIFT(R,TH,PH,X,Y)

```

C
DATA IFLAG /1/
IF(IFLAG.EQ.0) GOTO 100
IFLAG=0
XO=TH
YO=PH
X=0.
Y=.5
RETURN
C
100 CONTINUE
X=TH-XO
Y=PH-YO+.5
RETURN
END

```



```

SUBROUTINE BEARG1 (THT,STEEST,NST,NTAR,SLA,SLO)
C
C THIS PROGRAM GENERATES BEARINGS FROM A SET OF STATION
C AND TARGET POINTS. THE BEARINGS ARE GIVEN AS ANGLES FROM
C 0 TO 360 DEGREES, WHERE 0 DEGREES IS TRUE NORTH.
C
      REAL*8 ITL(12),LBL/' '
      REAL LB '/'
      COMMON /B/ TLA,TLO,ITL,XX,YY
      DIMENSION TLA(20),TLO(20),SLA(20),SLO(20),STDEV(20),
1DIST(20,20),STEEST(20,20),HD(20),XX(20),YY(20),X(20),
1THT(20,20),Y(20)
C
      TWOPI=2.*3.14159265
      PI=3.141592
      RADIUS=10800./PI
      RADDEG=PI/180.0
C
C READ TARGET POINTS
C
      READ (5,100) NTAR
      DO 1 I=1,NTAR
1 READ(5,102) TLA(I),TLO(I)
C
C READ STATION POINTS
C
      READ(5,100) NST
      DO 2 I=1,NST
2 READ(5,102) SLA(I),SLO(I),STDEV(I)
      WRITE(6,503)
      WRITE(6,500) (TLA(I),TLO(I),I=1,NTAR)
      WRITE (6,501) (SLA(I),SLO(I),STDEV(I),I=1,NST)
C
      DO 10 I=1,NST
C
C COMPUTE HEADING
C
      IF(I.EQ.NST) GOTO 5
      A=SLA(I+1)-SLA(I)
      TH=(SLA(I+1)+SLA(I))/2.
      B=(SLO(I+1)-SLO(I))*COS(TH*RADDEG)
      IF(ABS(B).LT.1.E-6) GOTO3
      HEAD=PI/2.-ATAN(A/B)
      IF(B.LT.0) HEAD=HEAD+PI
      GOTO 5
3 HEAD=0.
      IF(A.LT.0.) HEAD=PI
5 HD(I)=HEAD/RADDEG
C
C COMPUTE BEARING ANGLES AND DIST FROM I-TH STATION TO J-TH
C
      DO 10 J=1,NTAR
      IFLAG=2
      IF(SLO(I).GT.TLO(J)) IFLAG=1
      CA=ABS(SLO(I)-TLO(J))*RADDEG
      SB=(90.-TLA(J))*RADDEG
      SC=(90.-SLA(I))*RADDEG
      SA=ARCOS(COS(SB)*COS(SC)+SIN(SB)*SIN(SC)*COS(CA))
      CB=ARCOS((COS(SB)-COS(SA)*COS(SC))/(SIN(SA)*SIN(SC)))
      THT(I,J)=CB/RADDEG
      GOTO (4,7),IFLAG
4 THT(I,J)=360.-THT(I,J)
7 CONTINUE
C
C DIST CALCULATION
C
      DIST(I,J)=SA*RADIUS
C

```



```

C   STDEV AS A FUNCTION OF REL. BEARING
C
      ANG=HEAD-THT(I,J)*RADDEG
      STEEST(I,J)=STDEV(I)*(COS(ANG)**2+1.)
10  CONTINUE
C
C   OUTPUT RESULTS
C
      DO 54 K=1,4
      IL=K*6
      I1=IL-5
      WRITE (6,502)
      DO 53 I=I1,IL
      IF(I.GT.NST) GOTO 54
      WRITE (6,253) I
53  WRITE (6,255) (J,THT(I,J),DIST(I,J),J=1,NTAR)
54  CONTINUE
C
C   COMPUTE X-Y POSITIONS OF STATIONS
C
      DO 1000 I=1,NST
      CALL SHIFT(RADIUS,SLO(I),SLA(I),X(I),Y(I))
      XX(I)=X(I)
      YY(I)=Y(I)
1000 CONTINUE
      CALL DRAW(NST,X,Y,1,0,LB,ITL,.7,.7,0,6,2,2,6,6,0,L)
      CALL DRAW(NST,X,Y,2,1,LBL,ITL,.1,.05,0,9,2,2,9,10,0,L)
C
C   COMPUTE X-Y POSITIONS OF TARGETS
C
      DO 1001 I=1,NTAR
      CALL SHIFT(RADIUS,TLO(I),TLA(I),X(I),Y(I))
1001 CONTINUE
      CALL DRAW(NTAR,X,Y,2,2,LBL,ITL,.1,.1,0,9,2,2,9,10,0,L)
      RETURN
100  FORMAT(I2)
102  FORMAT(3F10.4)
253  FORMAT(///14X,'BEARING AND DISTANCE FROM STATION',I3,
&' TO TARGET INDICATED',//)
255  FORMAT(I16,2F20.2)
500  FORMAT(///14X,'TARLAT',4X,'TARLON',/(12X,2F10.5))
501  FORMAT(///14X,'STALAT',4X,'STALON',5X,'STDEV',//
&(12X,3F10.5))
502  FORMAT(1H1)
503  FORMAT(1H1,///14X,'COMPUTER RESULTS FOR SIX TARGETS'
&' AND 50% DETECTION')
      END
C
      SUBROUTINE CONVT (PH1,TH1,OB1,A,B,C)
C
      DATA PI/3.141593 /,RD/.0174533/
      PH=PH1*RD
      TH=TH1*RD
      OB=OB1*RD
      ST=SIN(TH)
      CT=COS(TH)
      SP=SIN(PH)
      CP=COS(PH)
      CO=COS(OB)
      SO=SIN(OB)
      DX=-SP*CO*CT-ST*SO
      DY=SO*CT-SP*CO*ST
      DZ=CP*CO
      X1=CP*CT
      Y1=CP*ST
      A=Y1*DZ-SP*DY
      B=SP*DX-X1*DZ
      C=X1*DY-Y1*DX
      RETURN
      END

```


SUBROUTINE CORREL (IC,NO,NT,OBS,X,GATE,ADDGT,KK,NSTA)

LIST OF VARIABLES

X(K,L) TRACK MATRIX
OBS(M,N) OBSERVATION MATRIX
C(I,J) CORRELATION MATRIX

NOTE SUBSCRIPTS ARE USED AS INDICATED ABOVE IF POSSIBLE

S(I) DISTANCE IN ANGLE OF OBSERVATION I FROM TRACK J
=101. IF UNCORRELATED
S(J) DISTANCE IN ANGLE OF OBSERVATION I FROM TRACK J
=101. IF UNCORRELATED

INTEGER C(20,20),SUM
COMMON /A/ NTR,NJ,TH1,TD,NFRST
DIMENSION X(5,20),GATE(5), OBS(5,20),S(20),ADDGT(20),
1KK(20),NJ(20,20)
DIMENSION TD(20,20),NFRST(20),IC(20),NTR(20),TH1(20)

DATA TDM/2.2918/,RD/.0174533/
IF(NSTA.GT.2) GOTO 11
DO 10 I=1,20
DO 10 J=1,20
10 C(I,J)=0
11 CONTINUE
NMN=NT
NMX=NT
IF(NO.GT.NT) NMX=NO
IF(NO.LT.NT) NMN=NO
DO 12 I=1,NMX
IC(I)=0
12 C(I,I)=0
GT=GATE(5)

WRITE (6,2)
WRITE (6,9)
WRITE (6,3) ((X(K,L),K=1,5),L=1,NT)
WRITE (6,5)
WRITE (6,9)
WRITE (6,3) ((OBS(M,N),M=1,5),N=1,NO)
DO 18 L=1,NT
GATE(5)=GT+ADDGT(L)
DO 18 N=1,NO
DO 17 K=1,5
TEMP=OBS(K,N)-X(K,L)
IF(ABS(TEMP).GT.GATE(K)) GOTO 18
17 CONTINUE
C(N,L)=1
18 CONTINUE
GATE(5)=GT

WRITE (6,6)
WRITE (6,7) (NTR(I),I=1,10)
DO 180 I=1,NO
180 WRITE (6,4) NJ(NSTA,I),(C(I,J),J=1,NT)

RULE 1

A TRACK CORRELATING WITH SEVERAL OBSERVATIONS REJECTS
ANY OBSERVATION HELD IN COMMON WITH AN OTHER TRACK, IF
THE COMMON OBSERVATION IS THE ONLY OBSERVATION
CORRELATING WITH THE OTHER TRACK

DO 19 J=1,NT


```

C
C
C      COMPUTE # OF OBSERVATIONS CORRELATED TO TRACK J
20  SUM=0
    DO 25 I=1,NC
25  SUM=C(I,J)+SUM
C
C
C      CHECK IF ONLY 1 OBSERVATION CORRELATED
    IF(SUM.LE.1) GOTO 19
    DO 119 I=1,NO
    IF(C(I,J).EQ.0) GOTO 119
    DO 21 JP=1,NT
    IF(C(I,JP).EQ.0) GOTO 21
C
C
C      COMPUTE # OF OBSERVATIONS CORRELATED TO TRACK JP
    SUM=0
    DO 23 IP=1,NO
23  SUM=SUM+C(IP,JP)
C
C
C      CHECK IF ONLY 1 OBSERVATION CORRELATED
    IF(SUM.NE.1) GOTO 21
    C(I,J)=0
    GOTO 20
21  CONTINUE
119 CONTINUE
19  CONTINUE
C
C
C      RULE 2
C
C      A TRACK CORRELATING WITH SEVERAL OBSERVATIONS, SOME OF
C      WHICH ARE NOT HELD IN COMMON WITH OTHER TRACKS, REJECTS
C      OBSERVATIONS HELD IN COMMON WITH OTHER TRACKS
    DO 31 J=1,NT
C
C
C      COMPUTE # OF OBSERVATIONS CORRELATED TO TRACK J
    SUM=0
    DO 32 I=1,NC
32  SUM=SUM+C(I,J)
C
C
C      CHECK IF ONLY 1 OBSERVATION CORRELATED
    IF(SUM.LE.1) GOTO 31
    DO 131 I=1,NO
    IF(C(I,J).EQ.0) GOTO 131
C
C
C      OBSERVATION I IS ONE OF SEVERAL OBSERVATIONS
C      CORRELATED TO TRACK J
C      COMPUTE # OF TRACKS CORRELATED TO OBSERVATION I
    SUM=0
    DO 35 JP=1,NT
35  SUM=SUM+C(I,JP)
C
C
C      CHECK WHETHER IN COMMON WITH OTHER TRACKS
    IF(SUM.GT.1) GOTO 131
C
C
C      REJECT OBSERVATIONS IN COMMON WITH OTHER TRACKS
    DO 231 K=1,NO
    IF(C(K,J).EQ.0) GOTO 231
    SUM=0
    DO 39 JP=1,NT
39  SUM=SUM+C(K,JP)
    IF(SUM.EQ.1) GOTO 231
    C(K,J)=0

```



```

231 CONTINUE
    GOTO 31
131 CONTINUE
31 CONTINUE

```

RULE 3

WHEN SEVERAL OBSERVATIONS CORRELATE WITH ONE TRACK, THE CLOSEST OBSERVATION IS CORRELATED TO THAT TRACK

```

DO 41 J=1,NT
SUM=0

```

COMPUTE # OF TRACKS CORRELATED.

```

DO 42 I=1,NC
42 SUM=SUM+C(I,J)
   IF(SUM.LE.1) GOTO 41

```

COMPUTE DISTANCE

```

DO 44 I=1,NC
S(I)=101.0
IF(C(I,J).EQ.0) GOTO 44
S(I)=ABS(X(5,J)-OBS(5,I))
44 CONTINUE

```

FIND CLOSEST OBSERVATION

```

NB=1
DO 46 IB=2,NO
IF(S(1).LE.S(IB)) GOTO 46
S(1)=S(IB)
NB=IB
46 CONTINUE
DO 49 I=1,NC
49 C(I,J)=0

```

RESET PROPER ELEMENT

```

C(NB,J)=1
41 CONTINUE

```

RULE 4

WHEN SEVERAL TRACKS CORRELATE WITH ONE OBSERVATION, THE OBSERVATION IS CORRELATED WITH THE CLOSEST TRACK

```

DO 51 I=1,NO

```

COMPUTE # OF TRACKS CORRELATED

```

SUM=0
DO 52 J=1,NT
52 SUM=SUM+C(I,J)
   IF(SUM.LE.1) GOTO 51

```

COMPUTE DISTANCE

```

DO 54 J=1,NT
S(J)=101.0
IF(C(I,J).EQ.0) GOTO 54
S(J)=ABS(X(5,J)-OBS(5,I))
54 CONTINUE

```

FIND CLOSEST TRACK.

```

NC=1
DO 56 JB=2,NT
IF(S(1).LE.S(JB)) GOTO 56

```



```

      S(1)=S(JB)
      NC=JB
56  CONTINUE
      DO 59 J=1,NT
59  C(I,J)=0
C
C      RESET PROPER ELEMENT
C
      C(I,NC)=1
51  CONTINUE
C
C      RULE 5
C
      IF AN OBSERVATION IS FOUND UNCORRELATED, AND AT THE SAME
      TIME THERE ARE TRACKS UNCORRELATED TO ANY OBSERVATION,
      THEN THIS OBSERVATION IS CORRELATED TO THE FIRST
      UNCORRELATED TRACK FOUND WITHIN THE GATE
C
      DO 515 I=1,NO
      DO 511 J=1,NT
      IF(C(I,J).EQ.1) GOTO 515
511  CONTINUE
      DO 514 J=1,NT
      DO 512 K=1,NO
      IF(C(K,J).EQ.1) GOTO 514
512  CONTINUE
      GATE(5)=GT+ADDGT(J)
      DO 513 K=1,5
      TEMP =OBS(K,I)-X(K,J)
      IF(ABS(TEMP).GT.GATE(K)) GOTO 514
513  CONTINUE
      C(I,J)=1
      GOTO 515
514  CONTINUE
515  CONTINUE
      GATE(5)=GT
C
      WRITE (6,8)
      WRITE (6,7) (NTR(I),I=1,10)
      DO 510 I=1,NO
510  WRITE (6,4) NJ(NSTA,I),(C(I,J),J=1,NT)
C
C      REARRANGE C-MATRIX TO DIAGONAL FORM
C
      L=0
C
      SEARCH DIAGONAL FOR ZEROES
C
      DO 62 I=1,NC
      M=I
      IF(C(I,I).NE.1) GOTO 63
      IC(I)=C(I,I)
62  CONTINUE
      IF(NO.EQ.NT) RETURN
      GOTO 75
C
C      SEARCH M-TH COLUMN FOR OFF DIAGONAL 1
C      INTERCHANGE CORRESPONDING ROWS
C
63  IF(M.EQ.NO) GOTO 66
      MM=M
      DO 65 M=MM,NO
      IF(M.GT.NMN) GOTO 66
      MP=M+1
      DO 65 I=MP,NO
      IF(C(I,M).EQ.0) GOTO 65
      C(I,M)=0
      C(M,M)=1
      DO 64 J=MP,NT
      IF(M.GE.NT) GOTO 640

```



```

        IF(C(M,J).EQ.0) GOTO 64
        C(M,J)=0
        C(I,J)=1
        GOTO 640
64  CONTINUE
640  CONTINUE
        TEMP=OBS(5,M)
        OBS(5,M)=OBS(5,I)
        OBS(5,I)=TEMP
65  CONTINUE
66  CONTINUE
C
C  SEARCH FOR UNCORRELATED OBSERVATIONS
C
        DO 69 I=1,NC
        IF(C(I,I).EQ.1) GOTO 69
        IP=I+1
        IF(IP.GT.NT) GOTO 675
        DO 67 J=IP,NT
        IF(C(I,J).EQ.1) GOTO 69
67  CONTINUE
675  CONTINUE
C
C  INITIALIZE NEW TRACK
C
        L=L+1
        NI=NT+L
        C(NI,NI)=1
        NFRST(NI)=NSTA
        KK(NI)=NSTA-1
        NTR(NI)=NJ(NSTA,I)
        TH1(NI)=OBS(5,I)
        DO 68 J=1,5
        X(J,NI)=OBS(J,I)
68  CONTINUE
        KKN1=KK(NI)
        TD(KKN1,NI)=TDM*SIN(X(5,NI)*RD)
69  CONTINUE
        IF(L.EQ.0) GOTO 700
        NTP=NT+1
        DO 70 I=NTP,NI
        DO 70 J=1,5
70  OBS(J,I)=X(J,I)
700  CONTINUE
C
C  CONTINUE TO REARRANGE THE C-MATRIX
C  SEARCH I-TH ROW FOR OFF DIAGONAL 1
C
        DO 74 I=1,NT
        IF(C(I,I).EQ.1) GOTO 74
        IP=I+1
        DO 73 J=IP,NT
        IF(C(I,J).EQ.0) GOTO 73
        DO 71 K=J,NT
        IF(C(J,K).EQ.0) GOTO 71
        C(I,K)=1
        C(J,K)=0
71  CONTINUE
        C(I,J)=0
        C(J,J)=1
        DO 72 K=1,5
        TEMP=OBS(K,J)
        OBS(K,J)=OBS(K,I)
72  OBS(K,I)=TEMP
73  CONTINUE
74  CONTINUE
C
C  SEARCH FOR MISSING DATA
C  SET KALMAN FILTER GAIN INDEX
C
75  NT=NT+L
        DO 77 I=1,NT

```



```

        IC(I)=C(I,I)
        IF(IC(I).EQ.0) GOTO 76
        X(4,I)=10
        GOTO 77
76      KK(I)=KK(I)+1
        X(4,I)=X(4,I)-1.
        IF(KK(I).GE.NSTA) KK(I)=NSTA-1
77      CONTINUE
        RETURN
2       FORMAT (///14X,'ORIGINAL MATRIX X OF PREDICTED TRACKS'
3       FORMAT (/,(12X,5F10.3))
4       FORMAT (I21,5X,20I3)
5       FORMAT (//,14X,18HINPUT OBSERVATIONS)
6       FORMAT (//,14X,'INITIAL CORRELATION MATRIX'/)
7       FORMAT (/30X,'TRACKS',//26X,10I3,//14X,'OBSERVATIONS'/)
8       FORMAT (/14X,24HFINAL CORRELATION MATRIX,/)
9       FORMAT (/14X,'FREQU',6X,'PRF',9X,'PW',7X,'CCUNT      DOA
        END

```

```

C       SUBROUTINE FIND (TLA,TLO,SIG,N,TH,SLA,SLO)
C
C       DIMENSION A(20),B(20),C(20),XS(3),XM(3),X1(200),X2(200
1X3(200),SLA(20),SLO(20),TH(20)
C
        DATA RD/.01744533/,R/3959./
        NM=N-1
        M=0
        DO 1 J=1,N
1       CALL CONVT (SLA(J),SLO(J),TH(J),A(J),B(J),C(J))
        DO 10 I=1,NM
        K=I+1
        DO 10 J=K,N
        M=M+1
        X3(M)=1.
        D=A(I)*B(J)-A(J)*B(I)
        X1(M)=(B(I)*C(J)-B(J)*C(I))/D
        X2(M)=(A(J)*C(I)-A(I)*C(J))/D
        E=R/SQRT(X1(M)**2+X2(M)**2+1.)
        X1(M)=X1(M)*E
        X2(M)=X2(M)*E
10      X3(M)=X3(M)*E
        DO 11 I=1,3
        XM(I)=0.
11      XS(I)=0.
        DO 12 I=1,M
        XM(1)=XM(1)+X1(I)
        XM(2)=XM(2)+X2(I)
        XM(3)=XM(3)+X3(I)
12      IF(N.EQ.2) GOTO 16
        DO 13 J=1,3
        XM(J)=XM(J)/M
        DO 14 I=1,M
        XS(1)=XS(1)+(X1(I)-XM(1))**2
        XS(2)=XS(2)+(X2(I)-XM(2))**2
        XS(3)=XS(3)+(X3(I)-XM(3))**2
14      DO 15 J=1,3
        XS(J)=XS(J)/M
        SIG=SQRT(XS(1)+XS(2)+XS(3))
15      CONTINUE
16      TLO=ATAN2(XM(2),XM(1))/RD
        SQ=SQRT(XM(1)**2+XM(2)**2)
        TLA=ATAN(XM(3)/SQ)/RD
        RETURN
        END

```


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<p>An airborne direction-finding technique capable of handling multiple emitters was developed. An adaptive gate size was introduced in the track correlations based on covariance relations of established tracks and observations. Track files were generated based upon the existence of various target parameters; i.e., frequency, pulse repetition frequency, pulse width and direction-of-arrival. To test the angular resolution capability of the filter, emitters in close proximity to each other with identical electronic characteristics were used in the simulation. Target locations are calculated in a cartesian coordinate system where the sphericity of the earth is taken into account, and with appropriate coordinate transformation computational simplicity is preserved.</p>			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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